

SECOND PROBLEM SESSION EXERCISES

(1) For given source f and observed data d let

$$J(m) = \frac{1}{2} \|d - \mathcal{F}(m)\|_2^2$$

with

$$u = \mathcal{F}(m) \iff m u_{tt} - \Delta u = f$$

and u satisfying appropriate boundary conditions. In class we saw how to compute

$$\frac{\delta \mathcal{F}}{\delta m}[m_0] m_1, \quad \text{and} \quad \frac{\delta J}{\delta m}[m_0] m_1.$$

(a) Compute

$$\left\langle \frac{\delta^2 \mathcal{F}}{\delta m^2}[m_0] m_1, m'_1 \right\rangle.$$

(b) Compute

$$\left\langle \frac{\delta^2 J}{\delta m^2}[m_0] m_1, m'_1 \right\rangle.$$