

The Chromatic Spectral Sequence

MRW 77

p prime in computations, assume p odd

$$BP_* = \mathbb{Z}_{(p)}[v_1, \dots] \quad |v_i| = 2(p^i - 1)$$

$$BP_* BP = BP_* [t_1, \dots] \quad |t_i| = 2(p^i - 1)$$

For $\forall x \in BP_*(x)$ is BP_* -comodule

$\mathcal{I}_n = (p, \dots, v_{n-1})$ only invariant prime ideals

$$\text{Ext}_{BP_* BP}^{H^*}(BP_*, BP_*) = H^*(\Sigma^+ BP_*, \partial)$$

$$\Sigma^+ BP_* = BP_* \underset{BP_*}{\otimes} BP_* BP \underset{BP_*}{\otimes} BP_* BP \dots \underset{BP_*}{\otimes} BP_* BP$$

$$d^0(m) = \sum m' \otimes m'' - m \otimes 1$$

$$BP_* \rightarrow BP_* \otimes BP_* BP$$

$$\begin{matrix} E^2 \\ E^1 \\ E^0 \end{matrix}$$

$$\text{ANSS} \quad E_2^* = H^*(BP_*)$$

want: understand everything in terms of H^0

[Novikov]: For $p \neq 2$, find $H^1 BP_*$

$$0 \rightarrow BP_* \xrightarrow{p^{n+1}} BP_* \rightarrow BP_*/p^{n+1} \rightarrow 0$$

$$\delta: H^0 \frac{BP_*}{p^{n+1}} \rightarrow H^1(BP_*)$$

$$V_1^{sp^n} \in H^0 \left(\frac{BP_*}{p^{n+1}} \right) \Rightarrow \delta(V_1^{sp^n}) \neq 0$$

"Greek letter element"

$$\alpha_{sp^n/n+1} = \delta(V_1^{sp^n}) \quad 1 \leq s \leq p+s$$

\approx order p^{n+1}

$$p=3$$

CS

$$N_n^0 = \cancel{BP_*} / I_n$$

$$M_n^0 = V_n^{-1} \cancel{BP_*} / I_n$$

$$0 \rightarrow N_r^S \rightarrow M_n^S \rightarrow N_n^{S+1} \rightarrow 0$$

$$M_n^{S+1} = V_{n+S+1}^{-1} N_n^{S+1}$$

$$N_n^S = \frac{BP_*}{(p_1, \dots, v_{n-1})} \quad , \quad N_p^S = \frac{BP_*}{(p_1, \dots, v_{n-1}, v_n^\infty)}$$

$$\varinjlim_m \frac{BP_*}{(\dots, v_n^m)}$$

Mult by

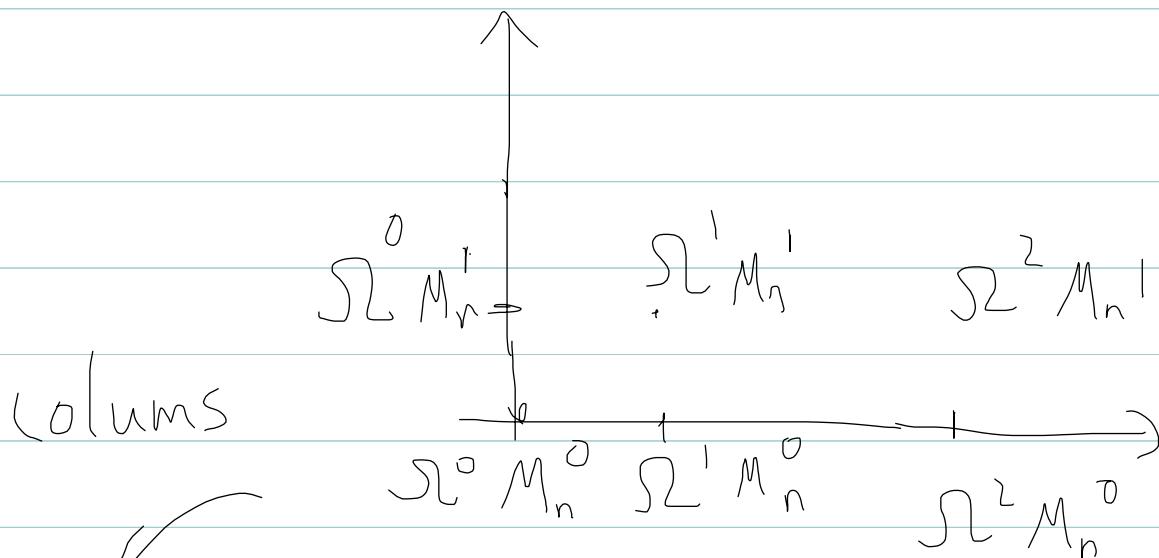
$$v_n$$

$$N_n^S = \frac{BP_*}{(p_1, \dots, v_{n-1}, v_n^\infty, \dots, v_{n+S-1}^\infty)}$$

$$M_n^S = V_{n+S}^{-1} \frac{BP_*}{(p_1, \dots, v_{n-1}, v_n^\infty, \dots, v_{n+S-1}^\infty)}$$

Chromatic
Cobar
Complex

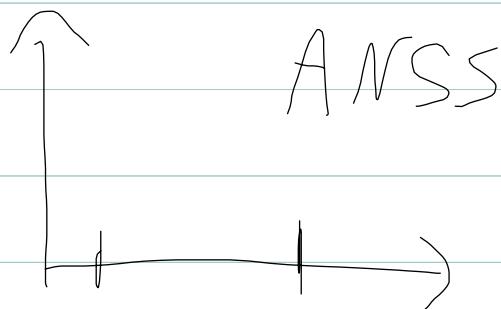
$$C_n^{\text{cy}} = \bigoplus_{\text{++} S=4} S^+ M_n^S$$



columns

Chromatic row cobar

$E_2 = E_\infty$



$$\mu^0 BP_* \xrightarrow{T^n} \mu^1 BP_* \xrightarrow{T^n} \dots \xrightarrow{T^n} \mu^S BP_*$$

$$\Rightarrow \mu^S BP_* \xrightarrow{T^n}$$

Now for the boundary maps:

Greek letter construction

$$H^0 N_0' \rightarrow H^1 BP_{\ast}$$

$$H^0 BP_{\ast} \xrightarrow{(p^\infty)} BP_{\ast} \xrightarrow{p^\infty} V_i^{Sp^n} \xrightarrow{} P^{n+1}$$

We define $H^t N_0^S \rightarrow H^{t+s} BP_{\ast}$

$$H^t N_0^S \xrightarrow{\delta_1} H^{t+1} N_0^{S-1} \xrightarrow{\delta_2} H^{t+s} BP_{\ast}$$

We had

$$N_0^{(S)} \rightarrow M_0^{S-1} \rightarrow N_0^S$$

$$N_0^{(S-2)} \rightarrow M_0^{S-2} \rightarrow N_0^{S-1}$$

$$+ = 0$$

$$m \left(\frac{v_s^{up}}{p^{io} v_1^{i_1} \dots v_n^{i_n}} \right) = \alpha^{(s)}_{u/i_1, \dots, i_p}$$

$$H^0 N_n^S = \frac{BP_*}{(p^\infty, \dots, v_{n-1}^\infty)}$$

$$0 \rightarrow M_{n+1}^{S-1} \xrightarrow{v_n} M_n^S \rightarrow M_n^S \rightarrow 0$$

$$\begin{array}{ccc} v_{n+S}^{-1} BP_* & \longrightarrow & v_{n+S}^{-1} BP_* \\ (p, v_1, \dots, v_n, v_{n+1}, \dots, v_{n+S}) & & (p, v_1, \dots, v_{n-1}, v_n) \end{array}$$

Bockstein SS

$$0 \rightarrow H^* M_{n+1}^{S-1} \rightarrow H^* M_n^S \rightarrow \dots$$

Process becomes inductive

$$M_{n+s}^0 \Rightarrow M_{n+s-1}^1 \Rightarrow \dots \Rightarrow M_n^s$$

Bockstein
Spectral
Sequence

$$M_{n+s}^0 = V_{n+s}^{-1} \xrightarrow{\text{BP}*} (p_1 V_1, \dots, V_{n-1}, V_n^\infty, \dots, V_{n+s-1}^\infty)$$

Change of ring iso:

$$M^* M_n^0 = F_x^+ \begin{pmatrix} K(n)_*, K(n)_* \\ K(n)_* K(n) \end{pmatrix}$$

computable

$$\mu^+ \mu^- \gamma$$

