The modal logic of forcing

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What are the most general principles in set theory relating forceability and truth? As with Solovay’s celebrated analysis of provability, both this question and its answer are naturally formulated with modal logic. We aim to do for forceability what Solovay did for provability. A set theoretical assertion $\psi$ is forceable or possible, if $\psi$ holds in some forcing extension, and necessary, if $\psi$ holds in all forcing extensions. In this forcing interpretation of modal logic, we establish that if ZFC is consistent, then the ZFC-provable principles of forcing are exactly those in the modal theory known as S4.2. The modal principles obeyed by forcing over any fixed model of set theory constitute a modal theory between S4.2 and S5, with both of these theories realized in various models of set theory. This is joint work with Benedikt Loewe.