18.S097 Introduction to Proofs
IAP 2015
Homework 2
Due: Friday, Jan. 9, 2015

Problem 1. *In this problem, we examine a second proof of the fact that there is no rational* $x \in \mathbb{Q}$ *such that* $x^2 = 2$.

(1) Suppose that the claim failed, i.e. that there exists $x \in \mathbb{Q}$ with $x^2 = 2$, and choose $\tilde{x} = |x|$ (so that $\tilde{x}^2 = 2$).
Show that there exists $q \in \mathbb{N} \setminus \{0\}$ such that

$$(\tilde{x} - 1)q \text{ is a non-negative integer.} \tag{1}$$

(2) Choose $q_* \in \mathbb{N} \setminus \{0\}$ as the smallest positive integer such that (1) holds, and define $q' = (\tilde{x} - 1)q_*$.

Show that:
(a) The inequalities $0 < q' < q_*$ hold (that is, show each of the inequalities $q' > 0$ and $q' < q_*$).
(b) The quantity $(\tilde{x} - 1)q'$ is a non-negative integer.

Observe that since this contradicts the minimality of q_*, no such $x \in \mathbb{Q}$ exists.