Introduction to Proofs
IAP 2015
In-class problems for day 3

Problem 4. *Use induction to show that the identity*

$$\frac{1-a^n}{1-a} = \sum_{k=0}^{n-1} a^k$$

holds for every $a \in \mathbb{R} \setminus \{1\}$ *and all* $n \in \mathbb{N}$.

Problem 5. *Show that for every* $n \in \mathbb{N}$, *if* $2^n - 1$ *is prime, then* n *is prime.*

Hint: Use the result of Problem 4 above.

Problem 6. *We showed in class that* $e := \sum_{k=0}^{\infty} \frac{1}{k!}$ *is irrational. Prove the stronger result that* e^2 *is irrational by using the series*

$$\frac{1}{e} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!}.$$

Question: Why do we say that this is a "stronger" result?
Hint: If $e^2 = \frac{a}{b}$, *write* $be = \frac{a}{e}$.