18.S097 Introduction to Proofs
IAP 2015
Homework 5
Due: Friday, Jan. 16, 2015

Choose and complete **one** of the following problems (you only have to do one!):

**Problem 1.** *Show that a function $f : \mathbb{R} \to \mathbb{R}$ is continuous if and only if for every open set $U \subset \mathbb{R}$, the set*

$$f^{-1}(U) = \{x \in \mathbb{R} : f(x) \in U\}$$

*is also open.*

**Problem 2.** *Let $(q_n)_{n\geq 1}$ be an enumeration of $\mathbb{Q} \cap (0,1)$ (so that $q_n \in \mathbb{Q} \cap (0,1)$ for each $n \geq 1$, and $\{q_n : n \geq 1\} = \mathbb{Q} \cap (0,1)$). Define*

$$A := \bigcup_{n\geq 1} \left( q_n - \frac{1}{2^{n+2}}, q_n + \frac{1}{2^{n+2}} \right).$$

*Show that $A^c := (0,1) \setminus A$ is not a set of measure zero.*