MATH 18.994 SYLLABUS FALL 2015

Lectures: MWF at 10:00 in E17-136.

Instructor: William Minicozzi, E17-344, 617-253-3299, minicozz@math.mit.edu.

Office Hours: My office hours are WF 9:00-10:00.

Text: *A Course in Minimal Surfaces*, by Colding and Minicozzi, AMS Graduate Studies in Mathematics, Volume 121.

Good PDE Reference (not required): *Partial Differential Equations*, by Evans, AMS Graduate Studies in Mathematics, Volume 19.

Presentations: Students will present several 40 minute lectures in groups. Students will also give a 20 minute presentation on their project.

Projects: Students will do a project in the subject area (topics will be suggested). They will write up a 10 page LaTex article on the project and then do a revision. They will also give a 20 minute presentation on the project in class.

Exams: There are NO EXAMS.

Grading: The presentations will count 50%, the project 40% and participation/attendance 10%.

Course Timeline: (Subject to change)

- 9/9 and 9/11 Introductory Lectures (Harmonic functions and Section 1.1).
- Student lectures 9/14, 16; Discussion 9/18.
- Student lectures 9/21, 23; Discussion 9/25.
- Student lectures 9/28, 30; Discussion plus project topics 10/2.
- Student lectures 10/5, 10/7; Discussion and **project topics due** 10/9.
- Student lectures 10/13, 14; Discussion 10/16.
- Student lectures 10/19, 21; Discussion and **partial draft due** 10/23.
- Student lectures 10/26, 28; Discussion 10/30.
- Student lectures 11/2, 4; Discussion and **projects due** 11/6.
- Student lectures 11/9; Discussion 11/13.
- Student lectures 11/16, 18; Discussion and **revised projects due** 11/20.
- Student lectures 11/23; Discussion 11/25.
- Project presentations 11/30, 12/2, 12/4 - and **final projects due**.
- Project presentations 12/7, 12/9.

This syllabus will be updated in the semester if there are any changes. The current version was prepared on August 27, 2015.

Student Lecture Topics:

(1) Chapter 1: Sections 1.2 and 1.3 - Submanifolds and First Variation.
(2) Chapter 1: Section 3 - Consequences of First Variation: Harmonicity, Convex Hull Property, Monotonicity and Meanvalue Inequalities.
(3) Chapter 1: Section 4 and Exercise 5 - Gauss Map and the Catenoid.
(4) Chapter 1: Section 6 - Weierstrass Representation.
(5) Strong maximum principle for elliptic equations (pages 326 to 333 in Evans).
(6) Chapter 1: Section 7 - Maximum Principle for Minimal Surfaces.
(7) Chapter 1: Section 8 up to (but not including) 8.3 - Second Variation and Stability.
(8) Chapter 1: 8.3 up to (not including) Proposition 1.39 - Stability and Bernstein's Theorem.
(9) Harnack inequality (pages 333-334 in Evans, PDE) and Fredholm Alternative (page 303 in Evans, PDE).
(10) Chapter 1: Proposition 1.39 - Characterization of Stability.
(11) Chapter 1: Appendix on the Bochner Formula.
(12) Chapter 2: Simons' equation - finish with (2.16).
(13) Chapter 2: Simons' inequality - start assuming (2.16).
(14) Chapter 2: Section 2 - Choi-Schoen curvature estimate.
(15) Chapter 2: Section 3 - Curvature and Area - finish with Section 3.2.
(16) Chapter 2: Section 3 - Curvature and Area - Section 3.3.
(17) Chapter 2: Section 4 - Schoen-Simon-Yau, part 1.
(18) Chapter 2: Section 5 - Schoen-Simon-Yau, part 2.
(19) Chapter 7: Section 2 - Hersch and Yang-Yau.
(20) Chapter 7: Sections $3, 4$ - Reilly Formula and Choi-Wang.

Some suggested Project Topics:

- The isoperimetric inequality for minimal surfaces (see, e.g., Chakerian, Proceedings of the AMS, volume 69, 1978).
- The Sobolev inequality (see Chapter 3).
- The Gauss-Bonnet Theorem (see Singer and Thorpe's book).
- Classify minimal surfaces in $\mathbf{R}^3$ whose Gauss map is one to one (see Theorem 9.4 in Osserman's book).
- Osserman's proof of the Nirenberg Conjecture: A complete minimal surface whose Gauss map omits an open set must be flat (Theorem 8.1 in Osserman's book).
- Mean curvature flow and Huisken's monotonicity formula (Huisken, JDG, volume 31, 1990).
- Almgren's Bernstein theorem in $\mathbf{S}^3$: the only minimal spheres are great spheres (Almgren, Annals of Math., volume 84, 1966).
- Constant mean curvature spheres and the Hopf problem (Hopf, Springer Lecture Notes in Math., volume 1000).