18.952
Homework Assignment 4
Due April 17

Problem 1, Section 4.1, Exercises vii – xii

Problem 2, Exercises:

1. In Section 4.1, Exercise viii, let $A_t \in S_n$ be a family of linear mappings depending smoothly on t and satisfying $A_t^2 = A_t$. Show that if $A_0 = A$ and $B = \frac{d}{dt} A_t(t = 0)$ then $BA + AB = B$.

2. Conclude from this that if V is the kernel of A and W the kernel of $I - A$, then B maps V into W and W into V.

3. Let $\langle v, w \rangle$ be the Euclidean inner product on $\mathbb{R}^n$. Show that since B is in S_n, $\langle Bv, w \rangle = \langle v, Bw \rangle$ for all $v \in V$ and $w \in W$, and conclude that B is determined by its restitution to V.

4. Show that if A is in $G(k, n)$, the tangent space to $G(k, n)$ at A is $\mathrm{Hom}(V, W)$.