

p. 229, Problem 10: Let S be a surface of revolution about the z -axis and $\varphi : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a rotation about the z -axis. We want to show that $\varphi : S \rightarrow S$ is an isometry. This however follows from the simple fact that $\varphi : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is an isometry and φ maps S to itself. More explicitly, we want to show that if w_1 and w_2 are any two tangent vectors at $p \in S$, then $\langle w_1, w_2 \rangle_p = \langle (d\varphi)_p(w_1), (d\varphi)_p(w_2) \rangle_{\varphi(p)}$. But $d\varphi = \varphi$, since φ is just given by a (rotation) matrix, and φ is an isometry. Thus, the previous equality holds.