

**Description**

These problems are related to the material covered in Lectures 20-23. Your solutions are to be written up in latex and submitted as a pdf-file with a filename of the form SurnamePset10.pdf via e-mail to drew@math.mit.edu by **noon** on the date due. Collaboration is permitted/encouraged, but you must identify your collaborators, and any references you consulted. If there are none, write “**Sources consulted: none**” at the top of your problem set. The first person to spot each non-trivial typo/error in any of the problem sets or lecture notes will receive 1-5 points of extra credit.

**Instructions:** First do the warm up problems, then pick 2 of the problems 1-4 to solve and write up your answers in latex, then complete the survey problem 5.

**Problem 0.**

These are warm up problems that do not need to be turned in.

- (a) Prove that for each integer  $n > 1$  there are infinitely many  $(\mathbb{Z}/n\mathbb{Z})$ -extensions of  $\mathbb{Q}$  ramified at only one prime.
- (b) Prove that for each integer  $n > 2$  there are no  $(\mathbb{Z}/n\mathbb{Z})^2$ -extensions of  $\mathbb{Q}$  ramified at only one prime. Why does this not contradict the fact that  $(\mathbb{Z}/p\mathbb{Z})^2$ -extensions of  $\mathbb{Q}_p$  exists for every  $p$ ?
- (c) In class we proved that in any finite extension of number fields infinitely many primes split completely. Must infinitely many primes remain inert?
- (d) Let  $K$  be a real quadratic field and let  $\infty$  denote the modulus supported on the real place of  $K$ . Show that if the fundamental unit of  $K$  has norm  $-1$  then  $\text{Cl}_K^\infty = \text{Cl}_K$  and otherwise  $\text{Cl}_K^\infty$  is larger than  $\text{Cl}_K$  by a factor of 2.

**Problem 1. Higher ramification groups (49 points)**

Let  $A$  be a complete DVR with finite residue field; its fraction field  $K$  is a nonarchimedean local field (Prop. 9.6). Let  $L$  be a finite Galois extension of  $K$ , let  $G := \text{Gal}(L/K)$ , and let  $B$  be the integral closure of  $A$  in  $L$ , with maximal ideal  $\mathfrak{q} = (\pi)$ . Fix  $\alpha \in B$  so that  $B = A[\alpha]$  (via Theorem 10.14), and let  $f \in A[x]$  be the minimal polynomial of  $\alpha$ .

The decomposition group  $D_{\mathfrak{q}}$  is equal to  $G$  (since  $\sigma(\mathfrak{q}) = \mathfrak{q}$  for all  $\sigma \in G$ ), and the inertia subgroup is  $I_{\mathfrak{q}} := \{\sigma \in G : \sigma(x) \equiv x \pmod{\mathfrak{q}} \text{ for all } x \in L\}$  with order equal to the ramification index  $e := e_{\mathfrak{q}}$ . For any integer  $i \geq -1$  define

$$G_i := \{\sigma \in G : \sigma(x) \equiv x \pmod{\mathfrak{q}^{i+1}} \text{ for all } x \in B\},$$

so that  $G_{-1} = G$  and  $G_0$  is the inertia subgroup. The group  $G_i$  is the  $i$ th *ramification group* of  $G$  (in the lower numbering). Define  $i_G : G \rightarrow \mathbb{Z} \cup \{\infty\}$  by  $i_G(\sigma) := v_{\mathfrak{q}}(\sigma(\alpha) - \alpha)$ .

- (a) Prove that  $G_i = \{\sigma \in G : i_G(\sigma) \geq i + 1\}$ , show that  $G_{i+1}$  is a normal subgroup of  $G_i$ , and show that the groups  $G_i$  are trivial for all sufficiently large  $i$ .

Recall that the different ideal  $\mathcal{D} := \mathcal{D}_{B/A}$  is equal to  $(f'(\alpha))$  and satisfies the bounds

$$e - 1 \leq v_{\mathfrak{q}}(\mathcal{D}) \leq e - 1 + v_{\mathfrak{q}}(e),$$

with  $e - 1 = v_{\mathfrak{q}}(\mathcal{D})$  if and only if  $v_{\mathfrak{q}}(e) = 0$ , by Proposition 12.23 and Theorem 12.26.

(b) Prove Hilbert's different formula:

$$v_{\mathfrak{q}}(\mathcal{D}) = \sum_{\sigma \neq 1} i_G(\sigma) = \sum_{i \geq 0} (\#G_i - 1).$$

Let  $U_0 := B^\times$  be the unit group of  $B$ , and for  $i > 0$  define

$$U_i := 1 + \mathfrak{q}^i = \{x \in U_0 : x \equiv 1 \pmod{\mathfrak{q}^i}\}.$$

- (c) Show that  $U_0/U_1 \simeq (B/\mathfrak{q})^\times$  and that for  $i > 0$  we have  $U_i/U_{i+1} \simeq \mathfrak{q}^i/\mathfrak{q}^{i+1}$  isomorphic to the additive group of  $B/\mathfrak{q}$ .
- (d) Fix  $i \geq 0$ . Show that for each  $\sigma \in G_i$  we have  $\sigma(\pi)/\pi \in U_i$ , and the map  $\sigma \mapsto \sigma(\pi)/\pi$  induces an injective group homomorphism  $\theta_i: G_i/G_{i+1} \hookrightarrow U_i/U_{i+1}$ .
- (e) Let  $i \geq 1$ . Show that for  $\sigma \in G_0$  and  $\tau \in G_i/G_{i+1}$  we have  $\theta_i(\sigma\tau\sigma^{-1}) = \theta_0(\sigma)^i\theta_i(\tau)$  (first show that both sides of this equality actually make sense, you may wish to invoke (c) when doing so). Then show  $\sigma\tau\sigma^{-1}\tau^{-1} \in G_{i+1} \Leftrightarrow \sigma^i \in G_1$  or  $\tau \in G_{i+1}$ .
- (f) Prove that for  $i, j \geq 1$ , if  $\sigma \in G_i$  and  $\tau \in G_j$  then  $\sigma\tau\sigma^{-1}\tau^{-1} \in G_{i+j+1}$ . Using this, show that the integers  $i \geq 1$  for which  $G_i \neq G_{i+1}$  are all congruent modulo  $p$ .

Now fix  $K = \mathbb{Q}_p$ , and let  $L/\mathbb{Q}_p$  be a finite Galois extension as above.

- (g) Show that if  $L/\mathbb{Q}_p$  is totally ramified of odd degree  $p$  then  $v_{\mathfrak{q}}(\mathcal{D}) = 2p - 2$ , and that if  $L/\mathbb{Q}_p$  is totally ramified of odd degree  $p^2$  then  $v_{\mathfrak{q}}(\mathcal{D}) = 3p^2 - p - 2$ .
- (h) Show that if  $L/\mathbb{Q}_p$  is totally ramified of odd degree  $p^2$  then  $G$  is cyclic. Conclude that no extension of  $\mathbb{Q}_p$  has Galois group isomorphic to  $(\mathbb{Z}/p\mathbb{Z})^3$ .

## Problem 2. The $p$ -adic logarithm (49 points)

The  $p$ -adic exponential is defined by

$$\exp(x) := \sum_{n \geq 0} \frac{x^n}{n!} \in \mathbb{Q}_p[[x]];$$

we may view it as a function on  $\mathbb{C}_p$  (the completion of the algebraic closure of  $\mathbb{Q}_p$  whose absolute value  $|\cdot|_p$  extends the  $p$ -adic absolute value on  $\mathbb{Q}_p$ ). For any power series  $\sum_{n \geq 0} a_n x^n$  over  $\mathbb{C}_p$ , we define its *radius of convergence*  $r$  in the usual way:

$$1/r := \limsup_{n \rightarrow \infty} |a_n|_p^{1/n}.$$

- (a) Show that for any power series  $f \in \mathbb{C}_p[[x]]$  with radius of convergence  $r$ , the series converges on  $|x|_p < r$ , diverges on  $|x|_p > r$ , and either converges for all  $x$  with  $|x|_p = r$ , or diverges for all  $x$  with  $|x|_p = r$ .

- (b) Show that  $v_p(n!) = \frac{n-s_n}{p-1}$ , where  $s_n$  is the sum of the digits of  $n$  when written in base  $p$ , and use this to compute the radius of convergence of  $\exp(x)$ .

We now define the  $p$ -adic logarithm by

$$\log(1+x) = \sum_{n \geq 1} (-1)^{n+1} \frac{x^n}{n}.$$

Let  $\mathfrak{m} := \{x \in \mathbb{C}_p : |x|_p < 1\}$  (this is the maximal ideal of the valuation ring of  $\mathbb{C}_p$ ).

- (c) Show that the power series defining  $\log(1+x)$  has radius of convergence 1; conclude that it gives a well defined function for  $x \in \mathfrak{m}$ .
- (d) Show that  $\log((1+x)(1+y)) = \log(1+x) + \log(1+y)$  for  $x, y \in \mathfrak{m}$ .
- (e) Let  $r$  be the radius of convergence of  $\exp$  you computed in (b). Prove that  $\log$  and  $\exp$  are inverse isomorphisms between the multiplicative group of the open disc of radius  $r$  about 1 and the additive group of the open disc of radius  $r$  about 0.

We are now in a position to fill in one of the missing details in our proof of the Kronecker-Weber theorem. Let  $\zeta_p$  denote a primitive  $p$ th-root of unity, let  $\pi = \zeta_p - 1$ , and let  $U_1$  denote the subgroup of  $\mathbb{Q}_p(\zeta_p)^\times$  congruent to 1 modulo  $\pi$ , and let  $U_1^p$  be the group of  $p$ th powers in  $U_1$ . We showed in lecture that the  $p$ -power map sends  $U_1$  to a subset  $U_1^p$  of  $\{u \equiv 1 \pmod{\pi^{p+1}}\}$ , but we actually used the fact that this map is surjective. With the  $p$ -adic logarithm we can easily invert the  $p$ -power map.

- (f) Show that the function  $f(x) := \exp\left(\frac{1}{p} \log x\right)$  maps each  $v \equiv 1 \pmod{\pi^{p+1}}$  to an element  $u \in U_1$  for which  $u^p = v$ , thus  $U_1^p = \{u \equiv 1 \pmod{\pi^{p+1}}\}$ .

Following Iwasawa, we now extend  $\log$  to a function on  $\mathbb{C}_p^\times$  by (arbitrarily) defining  $\log p = 0$ , and for  $x \in \mathfrak{m}$  and  $n \in \mathbb{Z}$  we define

$$\log(p^n(1+x)) := \log(1+x).$$

This extends  $\log$  to the subgroup  $G := p^{\mathbb{Z}}(1+\mathfrak{m})$  of  $\mathbb{C}_p^\times$ . For  $x \in \mathbb{C}_p^\times$  with  $x^n \in G$ , let

$$\log(x) := \frac{1}{n} \log(x^n).$$

- (g) Show that for every  $x \in \mathbb{C}_p^\times$  there is an integer  $n$  for which  $x^n \in G$  (thus our definition above covers all of  $\mathbb{C}_p^\times$ ).
- (h) Prove that  $\log: \mathbb{C}_p^\times \rightarrow \mathbb{C}_p$  is a homomorphism whose kernel is the subgroup of  $\mathbb{C}_p^\times$  generated by all roots of unity and all roots of  $p$ .

### Problem 3. The Frobenius density theorem (49 points)

Let  $L/K$  be a Galois extension of number fields of finite degree  $n$  with Galois group  $G := \text{Gal}(L/K)$ . Recall that for each unramified prime  $\mathfrak{p}$  of  $K$ , the *Frobenius class*  $\text{Frob}_{\mathfrak{p}}$  is the conjugacy class of the Frobenius elements  $\sigma_{\mathfrak{q}}$  for  $\mathfrak{q}|\mathfrak{p}$ .

The *Chebotarev density theorem* states that for any set  $C \subseteq G$  stable under conjugation (a union of conjugacy classes), the set of unramified primes  $\mathfrak{p}$  with  $\text{Frob}_{\mathfrak{p}} \subseteq C$  has Dirichlet density  $\#C/\#G$ .<sup>1</sup> In this problem you will prove the *Frobenius density theorem*, which says essentially the same thing, but with a different notion of conjugacy.

<sup>1</sup>It also has this natural density, but this was proved later.

**Definition.** Two elements  $g$  and  $h$  of a group  $G$  are *quasi-conjugate* if they generate conjugate subgroups  $\langle g \rangle$  and  $\langle h \rangle$ .

- (a) Show that quasi-conjugacy is an equivalence relation and that each quasi-conjugacy class in a group is a union of conjugacy classes.
- (b) Show that in the symmetric group  $S_n$ , each quasi-conjugacy class is actually a conjugacy class (so the Frobenius density theorem implies the Chebotarev density theorem in this case), but that this is generally not true for the alternating group  $A_n$ .
- (c) Suppose  $G$  is cyclic. For each  $d|n$ , let  $S_d$  be the set of primes  $\mathfrak{p}$  of  $K$  for which the primes  $\mathfrak{q}|\mathfrak{p}$  have inertia degree  $f_{\mathfrak{q}} = d$ . Prove that the set  $S_d$  has polar density  $\rho(S_d) = \phi(d)/[L:K]$  and conclude that infinitely many primes of  $K$  are inert in  $L$ .

Fix  $\sigma \in G$ , let  $K' = L^\sigma$  be its fixed field, let  $H = \langle \sigma \rangle \subseteq G$ , and let  $d = \#H$ . Recall that in any number field, a *degree-1 prime* is a prime whose absolute norm is prime. For each prime  $\mathfrak{p}$  of  $K$  (resp.  $K'$ ) that is unramified in  $L$ , let  $\overline{\text{Frob}}_{\mathfrak{p}}$  denote the quasi-conjugacy class in  $G$  (resp.  $H$ ) that contains the conjugacy class  $\text{Frob}_{\mathfrak{p}}$ .

- (d) Let  $S'$  be the set of degree-1 primes  $\mathfrak{p}'$  of  $K'$  for which  $\mathfrak{p} = \mathfrak{p}' \cap \mathcal{O}_K$  is unramified in  $L$  and for which  $\sigma \in \overline{\text{Frob}}_{\mathfrak{p}'}$ . Prove that  $S'$  has polar density  $\rho(S') = \phi(d)/d$ .
- (e) Let  $S$  be the set of unramified degree-1 primes  $\mathfrak{p}$  of  $K$  for which  $\sigma \in \overline{\text{Frob}}_{\mathfrak{p}}$ . Show that that map  $\mathfrak{p}' \mapsto \mathfrak{p}' \cap \mathcal{O}_K$  defines a surjective map  $\pi: S' \rightarrow S$ .
- (f) Show that the fibers of  $\pi$  all have cardinality  $[K':K]/c$ , where  $c$  is the number of distinct conjugates of  $H$  in  $G$ .
- (g) Show that  $S$  has polar density

$$\rho(S) = \frac{c\phi(d)}{[L:K]}$$

- (h) Prove that for any set  $C \subseteq G$  stable under quasi-conjugation the set of unramified primes  $\mathfrak{p}$  of  $K$  with  $\overline{\text{Frob}}_{\mathfrak{p}} \subseteq C$  has polar density  $\#C/\#G$ .

#### **Problem 4. The principal ideal theorem (49 points)**

The following theorem describes another remarkable (but not unique) property of the Hilbert class field.

**Theorem.** *Let  $K$  be a number field and let  $L$  be its Hilbert class field. Every  $\mathcal{O}_K$ -ideal generates a principal  $\mathcal{O}_L$ -ideal.*

This theorem was conjectured by Hilbert in 1900 and later reduced to a group theoretic question by Emil Artin that was finally proved by Furtwangler in 1930. One needs Artin reciprocity in order to prove it, so we will take this as given.

- (a) Show that the Hilbert class field  $M$  of  $L$  is a Galois extension of  $K$  and that  $\text{Gal}(L/K)$  is the maximal abelian quotient of  $\text{Gal}(M/K)$  (thus  $M/K$  is nonabelian unless  $M = L$ ).

Recall that for a finite group  $G$ , the maximal abelian quotient of  $G$  is  $G^{\text{ab}} := G/G'$ , the quotient of  $G$  by its commutator subgroup  $G' := \{ghg^{-1}h^{-1} : g, h \in G\}$ . If  $H$  is a (not necessarily normal) subgroup of  $G$ , there is a natural map  $V: G^{\text{ab}} \rightarrow H^{\text{ab}}$  called the *transfer map* (*Verlagerung* in German) which is defined as follows. Let  $S := \{g_1, \dots, g_n\}$  be a set of left coset representatives of  $H$  in  $G$ , define  $\phi: G \rightarrow S$  by  $g \in \phi(g)H$ , and put

$$V(g) := \prod_{i=1}^n \phi(gg_i)^{-1}gg_i.$$

- (b) Show that  $V$  induces a canonical homomorphism  $V: G^{\text{ab}} \rightarrow H^{\text{ab}}$  by showing that (i)  $V(g) \in H$  for all  $g \in G$ , (ii) the induced map  $G \rightarrow H^{\text{ab}}$  is a homomorphism with  $G'$  in its kernel, (iii) this homomorphism does not depend on the choice of  $S$ .

The Artin map  $\psi_{L/K}$  induces an isomorphism  $\text{Cl}(K) \xrightarrow{\sim} \text{Gal}(L/K)$ , and the Artin map  $\psi_{M/L}$  induces an isomorphism  $\text{Cl}(M) \xrightarrow{\sim} \text{Gal}(M/L)$ . The groups  $\text{Gal}(L/K)$  and  $\text{Gal}(M/L)$  are both abelian, hence equal to their maximal abelian quotients, We have a diagram of group homomorphisms

$$\begin{array}{ccc} \text{Cl}(K) & \xrightarrow{\sim} & \text{Gal}(L/K) = \text{Gal}(M/K)^{\text{ab}} \\ \downarrow \pi & & \downarrow V \\ \text{Cl}(L) & \xrightarrow{\sim} & \text{Gal}(M/L) = \text{Gal}(M/L)^{\text{ab}} \end{array}$$

where  $\pi$  is the map  $[\mathfrak{a}] \mapsto [\mathfrak{a}\mathcal{O}_L]$ . To prove the principal ideal theorem we need to show (1) this diagram commutes, and (2) the image of  $V$  on the RHS is trivial.

Put  $G := \text{Gal}(M/K)$  and  $H := \text{Gal}(M/L)$  so that  $G/H \simeq \text{Gal}(L/K) = \text{Gal}(M/K)^{\text{ab}}$  (thus  $H = G'$ , so it is not unreasonable to think  $V: G^{\text{ab}} \rightarrow H^{\text{ab}}$  should be trivial).

Let  $\mathfrak{p}$  be a prime of  $K$  and let  $\mathfrak{p}\mathcal{O}_K = \mathfrak{q}_1 \cdots \mathfrak{q}_r$  be its factorization into primes of  $L$ . Fix a prime  $\mathfrak{r}|\mathfrak{q}|\mathfrak{p}$  of  $M$  above a prime  $\mathfrak{q}|\mathfrak{p}$  of  $L$  (say  $\mathfrak{q} = \mathfrak{q}_1$ ), define  $\tau_i$  by  $\tau_i(\mathfrak{q}_i) = \mathfrak{q}$ , let  $D \subseteq G$  be the decomposition group of  $\mathfrak{r}$  over  $K$ , and let  $g := \sigma_{\mathfrak{r}} \in G$  be the Frobenius element at  $\mathfrak{r}$ .

- (c) Show that the image of  $\mathfrak{p}$  in  $H$  under  $\psi_{M/L} \circ \pi$  is  $\prod_i \psi_{M/L}(\mathfrak{q}_i)$  and that the double cosets  $D\tau_i H$  are distinct and cover  $G$ .
- (d) Define  $g_{ij} := g^j \tau_i$  for  $0 \leq j < m$ , where  $m$  is the order of  $\psi_{L/K}(\mathfrak{p})$ , and show that  $S := \{g_{ij}\}$  is a unique set of left coset representatives for  $H$ .
- (e) Using  $S := \{g_{ij}\}$  to define the maps  $\phi$  and  $V$  above, show that for each  $\mathfrak{q}_i|\mathfrak{p}$  we have

$$\psi_{M/L}(\mathfrak{q}_i) = \prod_{j=0}^{m-1} \phi(gg_{ij})^{-1}gg_{ij}$$

and conclude that the diagram above commutes.

- (f) Let  $\mathbb{Z}[G]$  be the (noncommutative) group algebra of  $G$  (formal sums  $\sum n_g[g]$  over  $\{[g] : g \in G\}$  with  $[g][h] = [gh]$ ). Let  $I_G$  be the *augmentation ideal* of sums  $\sum n_g[g]$  for which  $\sum n_g = 0$ , and let

$$\delta: H/H' \rightarrow (I_H + I_G I_H)/(I_G I_H)$$

be the homomorphism that sends the class of  $h \in H$  in  $H/H'$  to the class of  $[h] - 1$  in  $(I_H + I_G I_H)/(I_G I_H)$  (here 1 is the identity in  $\mathbb{Z}[G]$ ). Prove that  $\delta$  is an isomorphism (hint: show that  $\{[g]([h] - 1) : g \in S, h \in H\}$  is a basis for  $I_H + I_G I_H$  as a  $\mathbb{Z}$ -module).

(g) Prove that the diagram

$$\begin{array}{ccc} G/G' & \xrightarrow{V} & H/H' \\ \downarrow \delta & & \downarrow \delta \\ I_G/I_G^2 & \xrightarrow{\varphi} & (I_H + I_G I_H)/(I_G I_H) \end{array}$$

commutes, where  $\varphi(x) = x([g_1] + \cdots + [g_n])$ .

(h) Prove that if  $G$  is a finite group and  $H = G'$  then  $V : G^{\text{ab}} \rightarrow H^{\text{ab}}$  has trivial image (hint: quotient by  $H'$  to reduce to the case that  $H$  is abelian then write  $G/H = G/G'$  as a product of cyclic groups and go from there; if you get stuck feel free to consult [1, Theorem VI.7.6] for further details on how to proceed).

### Problem 5. Class fields of $\mathbb{Q}$ (49 points)

- (a) Show that the ray class fields of  $\mathbb{Q}$  consist of the cyclotomic fields  $\mathbb{Q}(\zeta_m)$  and their maximal real subfields  $\mathbb{Q}(\zeta_m)^+ := \mathbb{Q}(\zeta_m + \zeta_m^{-1})$ . For integers  $m > 2$  show that  $[\mathbb{Q}(\zeta_m) : \mathbb{Q}(\zeta_m)^+] = 2$  and that  $\mathbb{Q}(\zeta_m)$  is *totally complex* (its archimedean places are all complex) while  $\mathbb{Q}(\zeta_m)^+$  is *totally real* (its archimedean places are all real).
- (b) Solve the abelian inverse Galois problem over  $\mathbb{Q}$  by showing that every finite abelian group is isomorphic to the Galois group of an extension of  $\mathbb{Q}$ .
- (c) Does (b) still hold if we restrict to totally real extensions of  $\mathbb{Q}$ ?

Recall that the *conductor* of a congruence subgroup is the minimal modulus that appears in its equivalence class; it follows from the Artin reciprocity law that the conductor of the corresponding abelian extension  $L/K$  is the minimal modulus  $\mathfrak{m}$  of a ray class field  $K(\mathfrak{m})$  that contains  $L$ . Our next goal is to determine the set of conductors for abelian extensions of  $K = \mathbb{Q}$ , but we will initially work in greater generality.

Let  $\mathfrak{p}_2$  denote a prime of  $K$  of absolute norm  $N(\mathfrak{p}_2) = 2$  (if one exists) and suppose  $\mathfrak{m}$  is the conductor of some congruence subgroup for  $K$ .

- (d) Show that if  $\mathfrak{m}_0$  is trivial then  $\#\mathfrak{m}_\infty \neq 1$ .
- (e) Show that if  $\mathfrak{p}_2 | \mathfrak{m}$  then  $\mathfrak{p}_2^2 | \mathfrak{m}$ .
- (f) Show that if  $\mathfrak{m} = \mathfrak{p}_2^2$  then  $\mathfrak{p}_2$  is ramified in  $K/\mathbb{Q}$ .
- (g) Show that if  $\#\mathfrak{m}_\infty = 0$  then  $N(\mathfrak{m}_0) \neq 3$ .
- (h) Show that the only moduli that are not conductors of an abelian extension of  $\mathbb{Q}$  are those ruled out by (d)–(g), namely:  $\infty$ , (3), (4),  $(m)$  and  $(m)_\infty$ , for all  $m \equiv 2 \pmod{4}$ .

## Problem 6. Survey

Complete the following survey by rating each problem you attempted on a scale of 1 to 10 according to how interesting you found it (1 = “mind-numbing,” 10 = “mind-blowing”), and how difficult you found it (1 = “trivial,” 10 = “brutal”). Also estimate the amount of time you spent on each problem to the nearest half hour.

	Interest	Difficulty	Time Spent
Problem 1			
Problem 2			
Problem 3			
Problem 4			
Problem 5			

Please rate each of the following lectures that you attended, according to the quality of the material (1=“useless”, 10=“fascinating”), the quality of the presentation (1=“epic fail”, 10=“perfection”), the pace (1=“way too slow”, 10=“way too fast”, 5=“just right”) and the novelty of the material to you (1=“old hat”, 10=“all new”).

Date	Lecture Topic	Material	Presentation	Pace	Novelty
11/20	Ray class groups and fields				
11/25	Global class field theory				
11/27	Artin reciprocity				

Please feel free to record any additional comments you have on the problem sets and the lectures, in particular, ways in which they might be improved.

## References

- [1] Jürgen Neukirch, *Algebraic number theory*, Springer-Verlag, 1999.