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1 Lecture 8

Ex 8.1 Let $\mathfrak{g} = gl_n(\mathbb{F})$, char$\mathbb{F} = 2$. Let $\mathfrak{h} = \left\{ \text{matrices of the form } \begin{pmatrix} \lambda & \cdots & * \\ & \ddots & \\ 0 & \cdots & \lambda \end{pmatrix} \right\}$. Then this is a maximal nilpotent subalgebra, but not a Cartan subalgebra.

Ex 8.2: Any rank 1 3-dim algebra is isomorphic to one of the following algebras with basis h, a, b:

1. $[h, a] = a, [h, b] = a + b, [a, b] = 0$
2. $[h, a] = a, [h, b] = \lambda b \neq 0, [a, b] = 0.$
3. $[h, a] = a, [h, b] = -b, [a, b] = h$

Ex 8.3: Show that all lie algebras in 8.2 are non-isomorphic. Show that the first and second are solvable, and the third is isomorphic to $\mathfrak{sl}_2(\mathbb{F})$, which is not solvable.