Lecture 3 Exercises

3.1

Show $Z(gl_n(\mathbb{F})) = \mathbb{F}I_n$ and $Z(sl_n(\mathbb{F})) = 0$ if *char* $\mathbb{F} \nmid n$.

3.2

Let dim $\mathfrak{g} < \infty$. Show dim $Z(\mathfrak{g}) \neq$ dim $\mathfrak{g} - 1$.

3.3

Classify all finite dimensional Lie algebras for which dim $Z(\mathfrak{g}) =$ dim $\mathfrak{g} - 2$. Let $\dim Z(\mathfrak{g}) = n$ and show either $Z(\mathfrak{g}) = Ab_{n-3} \bigoplus \text{Heis}_3$ or $Z(\mathfrak{g}) = Ab_{n-2} \bigoplus \mathfrak{h}$ where $\mathfrak{h}$ is the two-dimensional non-abelian Lie algebra. We define Heis_{2n+1} to be the Lie algebra with basis $\{p_i, q_i, c\}$ where $[p_i, q_j] = \delta_{i,j}c = -[q_j, p_i]$ and all other bracketed pairs are 0.

3.4

Show if dim $V < \infty$, then A is nilpotent if and only if all eigenvalues are zero.

3.5

Construct in $sl_3(\mathbb{F})$ a two-dimensional subspace consisting of nilpotent matrices, which do not have a common eigenvector.