Exercise 14.1. Prove that: $\Delta \subset V \setminus \{0\}$ is an indecomposable set if and only if for any $\alpha, \beta \in \Delta$, there exists a sequence $\gamma_1, \gamma_2 \dots \gamma_s$ such that $\alpha = \gamma_1$, $\beta = \gamma_s$ and $\gamma_i + \gamma_{i+1} \in \Delta$ for $i = 1 \dots s-1$.

Also for any $\Delta \subset V \setminus \{0\}$, construct its canonical decomposition into a disjiont union of indecomposable sets.

Exercise 14.2. Recall that a semisimple Lie algebra $\mathfrak{g}$ is a direct sum of $\bigoplus_{j=1}^{N} s_j$ where s_j are simple Lie algebras.

Prove that this decomposition is unique up to permutation of the summands and prove that any ideal of $\mathfrak{g}$ is a subsum of this sum.

Exercise 14.3. The argument from class fails if $char\mathbb{F}$ divides n. $\mathfrak{sl}_n(\mathbb{F})$ contains a non-trivial abelian ideal, $Z(\mathfrak{sl}_n(\mathbb{F})$, as $I_n \in Z(\mathfrak{sl}_n(\mathbb{F})$. How does the argument fail?