## Problem Set 9

**Due:** Tue, November 22 at 11 AM in the pset boxes outside room 4–174

**NO COLLABORATION**

1) An $R$–module $M$ is called **faithful** if $\mathrm{Ann}(M) = (0)$. Show that if a ring admits a faithful Noetherian module, then the ring is Noetherian.

2) Let $R$ be an arbitrary ring. Show that if an $R$–module $M$ is finitely generated and Artinian, then it is Noetherian.

3) Let $R$ be a Noetherian ring. Prove that if $M$ is a finite length $R$–module, then the ring $R/\mathrm{Ann}(M)$ is Artinian.

4) Prove that if $R$ is a Noetherian ring, then so is $R[[x]]$.