Problem Set 7

Due: Tue, November 1 at 11 AM in the pset boxes outside room 4–174

NO COLLABORATION

1) For an arbitrary prime power q, give an example of a polynomial $f \in \mathbb{F}_q[x, y]$ such that for every $a \in \mathbb{F}_q$, the ring $\mathbb{F}_q[x, y]/(f)$ is NOT a finitely generated module over the ring $\mathbb{F}_q[x + ay]$.

(As a side remark, this means that our proof of Noether normalization fails in the case when the ground field is finite. The result is still true, though)

2) Find the normalization (i.e. the integral closure of R in its fraction field) of the ring $R = \mathbb{C}[x, y]/(x^3 - y^4)$.

3) Suppose that $R \subset R'$ are commutative rings, and that a certain polynomial $f \in R[x]$ factors as a product of two MONIC polynomials in $R'[x]$. Prove that the coefficients of the two factors are integral over R.

4) Prove that the integral closure of $\mathbb{Z}$ in $\mathbb{C}$ is NOT finitely generated as a $\mathbb{Z}$–module.