Problem Set 2

Due: Tue, September 20 at 11 AM in the pset boxes outside room 4–174

NO COLLABORATION

1) A **local ring** is a ring with a single maximal ideal. Prove that if $I \subset R$ is a maximal ideal such that $1 + x$ is a unit for all $x \in I$, then R is a local ring. Does this implication still hold if I is not assumed to be maximal?

2) Describe the topological space Spec $\mathbb{R}[x]$ endowed with the Zariski topology (don't state any fact without an argument)

3) Suppose $e_1, ..., e_k$ are idempotents in a ring R. Show that the ideal $(e_1, ..., e_k)$ is not only principal, but is generated by a single idempotent.

4) Use Zorn's Lemma to prove that any prime ideal contains a minimal prime ideal (note that you can't just use the 0 ideal, since it is not always prime).