18.701 Problem Set 8

Because of the quiz on November 7, this assignment is due Wednesday, November 14.

1. Let $G$ be the group $GL_3(F_2)$. Its Class Equation, which is $168 = 1 + 21 + 42 + 56 + 24 + 24$, was computed in the previous assignment.

   (i) Determine the numbers of $p$-Sylow subgroups with $p = 2, 3, 7$.

   (ii) Determine the orders of the elements of $G$ and the number of elements of each order.

2. Chapter 7, Exercise 8.6. (groups of order 55)

3. Use the Todd-Coxeter Algorithm to determine the order of the group generated by two elements $x, y$.

   (a) with relations $x^3 = 1$, $y^3 = 1$, and $xyx = yxy$.

   (b) with relations $x^3 = 1$, $y^4 = 1$, and $xyxy = 1$.

   Be careful. Any error will collapse the group. I recommend practicing on some simpler relations first.

4. Chapter 7, Exercise M.1. Classify groups generated by two elements $x, y$ of order two.

   The element $z = xy$ is useful for this.