18.701 Practice Quiz 3

You are expected to prove your assertions, but you may state and use without proof results from lectures or from the assigned reading, unless you are asked to prove them here.

The questions are of equal value.

1. Prove that every group of order 105 contains at least one proper normal subgroup.

2. Let $W$ be the subspace of $V = \mathbb{R}^3$ spanned by the vectors $w_1 = (1, 2, 1)^t$ and $w_2 = (1, 1, 0)^t$. Determine the orthogonal projection $\pi : V \to W$, with respect to the standard basis of $V$, and with the dot product form on $V$.

3. What does the Spectral Theorem tell us about a $2 \times 2$ hermitian matrix whose characteristic polynomial has a double root?

4. What are the possible eigenvalues of a matrix that is
   (i) positive definite hermitian?
   (ii) unitary?

5. Let $G$ be the group of real matrices of the form $\begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix}$. Determine the one-parameter groups in $G$. 