18.701 Practice Quiz 1

You are expected to justify your assertions, but you may use without proof results from class and from the text.

The problems are of equal value.

1. Let $G$ be a cyclic group of order 15. How many of the elements of $G$ are generators for the group?

2. Let $\varphi$ denote the homomorphism $\mathbb{R}^+ \to \mathbb{C}^\times$ defined by $\varphi(x) = e^{ix}$. Determine the kernel and the image of $\varphi$.

3. Decide whether the permutation $(1\,2\,3\,4)(2\,3\,4\,5)$ is odd or even.

4. How many elements of order 2 does the symmetric group $S_4$ contain?

5. Let $G \xrightarrow{\varphi} C_6$ be a surjective homomorphism from a group $G$ to a cyclic group of order 6, and let $K$ be the kernel of $\varphi$. How many subgroups of $G$ contain $K$?

6. (30 points) Let $H$ be a subgroup of of a group $G$.

   (i) Define the index $[G : H]$ of $H$ in $G$.

   (ii) Suppose that the index is $n$. Prove that for every element $x$ of $G$ there is an integer $k$ with $1 \leq k \leq n$, such that $x^k$ is in $H$. 