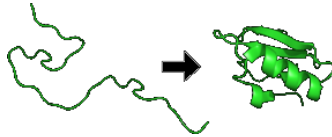
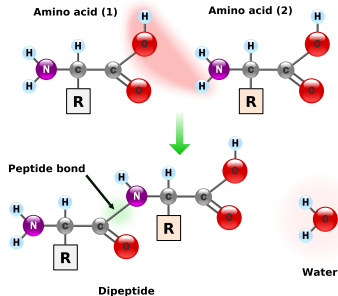


Protein Structure Prediction



- Protein = chain of amino acids (AA)
- aa connected by peptide bonds



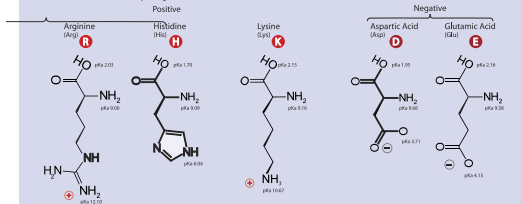
Amino Acids



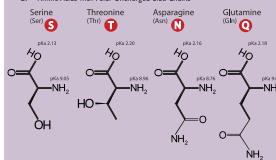
Twenty-One Amino Acids

⊕ Positive
⊖ Negative
• Side chain charge at physiological pH 7.4

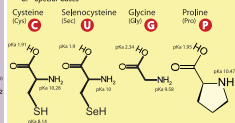
A. Amino Acids with Electrically Charged Side Chains



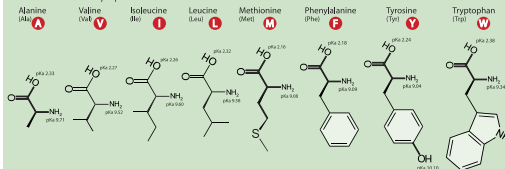
B. Amino Acids with Polar Uncharged Side Chains



C. Special Cases

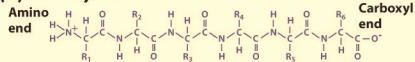


D. Amino Acids with Hydrophobic Side Chains

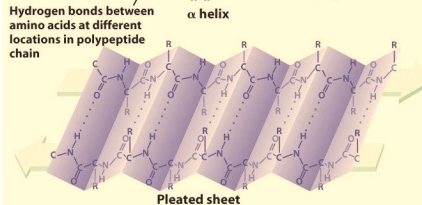
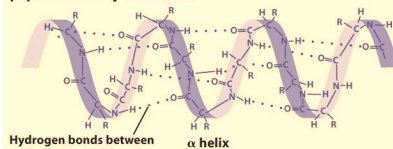


Levels of structure

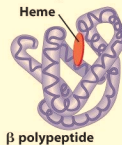
(a) Primary structure



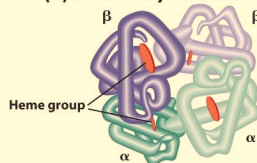
(b) Secondary structure



(c) Tertiary structure



(d) Quaternary structure



Protein Structure Prediction



Christian Anfinsen, 1961:

denatured RNase refolds into functional state (in vitro)

⇒ no external folding machinery

⇒ Anfinsen's dogma/thermodynamic hypothesis:

all information about native structure is in the sequence
(at least for small globular proteins)

native structure = minimum of the free energy

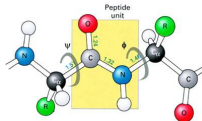
- unique
- stable
- kinetically accessible

Levinthal's Paradox, 1969

Cyrus Levinthal: protein folding is not trial-and-error

Thought experiment:

- protein with 100 peptide bonds (101 aa)
- assume 3 states for each of the 200 phi and psi bond angles

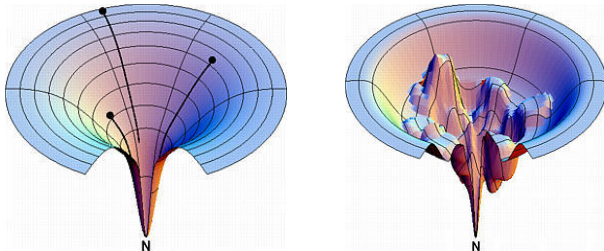


- $\Rightarrow 3^{200} \approx 10^{95}$ conformations
- assuming one quadrillion samples per second, still over 60 orders of magnitude longer than the age of the universe

BUT: proteins fold in milliseconds to seconds

PARADOX

Principles of Folding 'Essentially' Understood



Folding Funnel resolves Levinthal's Paradox

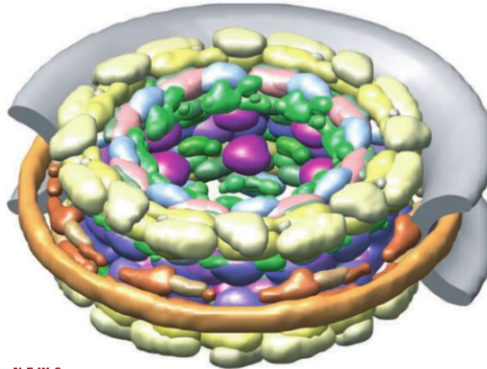
Driving forces:

- hiding of non-polar groups away from water
- close, nearly void-free packing of buried groups and atoms
- formation of intramolecular hydrogen bonds by nearly all buried polar atoms

Hydrophobic effect · Van-der-Waals · Electrostatic

August 8th, Science: problem solved?

Challenges in Theoretical Chemistry



NEWS

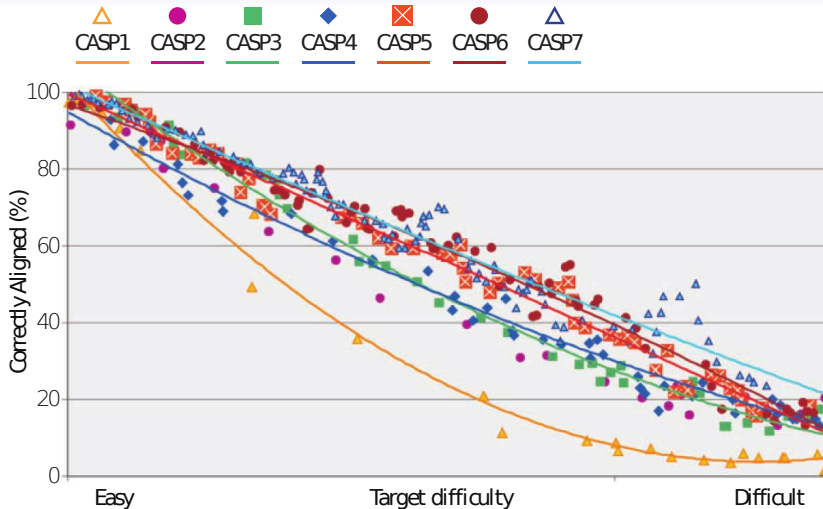
Problem Solved* (*sort of)



Robert F. Service. Problem solved* (*sort of). Science, 2008.

[this and some following slides inspired by Jinbo Xu, Jerome Waldispühl]

Increasing Accuracy of Predictions: Slowly but Steadily



Steady rise. Computer modelers have slowly but steadily improved the accuracy of the protein-folding models.

Distance between 3D structures

RMSD = Root Mean Square Deviation

Compares two vectors of coordinates (here, coordinates of atoms in protein conformations). Yields distance between conformations.

$$\begin{aligned}\text{RMSD}(v, w) &= \sqrt{\frac{1}{n} \sum \|v_i - w_i\|^2} \\ &= \sqrt{\frac{1}{n} \sum (v_{ix} - w_{ix})^2 + (v_{iy} - w_{iy})^2 + (v_{iz} - w_{iz})^2}\end{aligned}$$

RMSD depends on orientation;

it is applied to superimposed structures, or after minimizing over rotations/translations (Kabsch algorithm)

CASP/CAFASP

- CASP:

Critical
Assessment of
Structure
Prediction



CASP
Predictor

- CAFASP:

Critical
Assessment of
Fully
Automated
Structure
Prediction



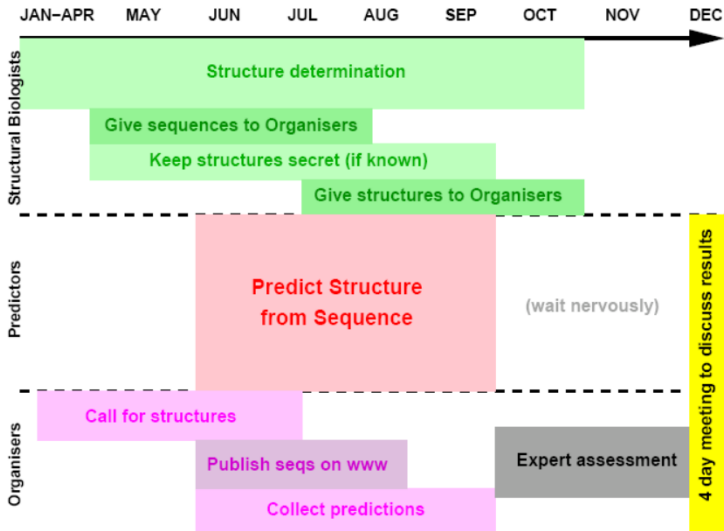
CAFASP
Predictor

1. Won't get tired
2. High-throughput

CASP/CAFASP

- Public
 - Organized by structure community
 - Evaluated by the unbiased third-party
 - Held every two years
- Blind:
 - Experimental structures to be determined by structure centers after competition
- Drawback: <100 targets
 - Blindness
 - Some centers are reluctant to release their structures

CASP/CAFASP Schedule



Test Protein Category

- New Fold (NF) targets
 - No similar fold in PDB
- Homology
 - Modeling (HM) targets
 - Easy HM: has a homologous protein in PDB
 - Hard HM: has a distant homologous protein in PDB
 - Also called Comparative Modeling (CM) targets
- Fold Recognition (FR) targets
 - Has a similar fold in PDB

Protein Structure Prediction

- Stage 1: Backbone Prediction
 - Ab initio prediction
 - Homology modeling
 - Protein threading
- Stage 2: Loop Modeling
- Stage 3: Side-Chain Packing
- Stage 4: Structure Refinement

Protein Structure Prediction

- Stage 1: Backbone Prediction
 - **Ab initio prediction**
 - Homology modeling
 - Protein threading
- Stage 2: Loop Modeling
- Stage 3: Side-Chain Packing
- Stage 4: Structure Refinement

Ab-initio Prediction: Sampling the global conformation space

- Lattice models / Discrete-state models
- Molecular Dynamics
- Fragment assembly
from pre-set library of 3D motifs (=fragments)

Ab-initio Prediction: Sampling the global conformation space

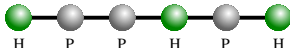
- Lattice models / Discrete-state models
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- Fragment assembly
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Lattice Models: The Simplest Protein Model

The HP-Model (Lau & Dill, 1989)

- model only hydrophobic interaction
 - alphabet $\{H, P\}$; H/P = hydrophobic/polar
 - energy function favors HH-contacts
- structures are discrete, simple, and 2D
 - model only backbone (C- α) positions
 - structures are drawn on a square lattice \mathbb{Z}^2 without overlaps: Self-Avoiding Walk

Example

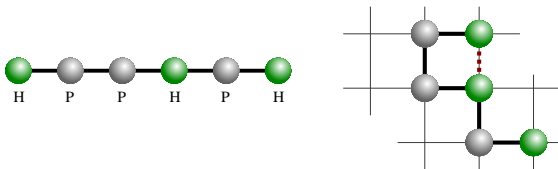


Lattice Models: The Simplest Protein Model

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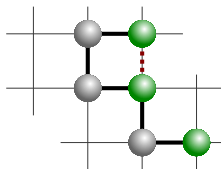
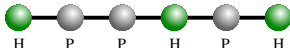


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Example



HH-contact

Lattice Models: Discrete Structure Space

Structure space of a sequence = set of possible structures

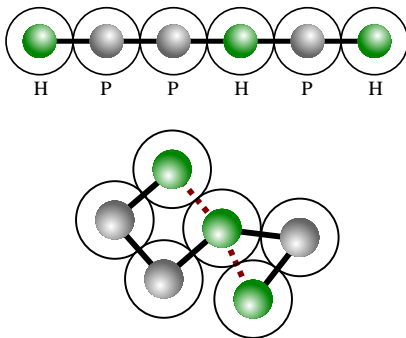
Lattices

- Lattice discretizes the structure space
- Structures can be enumerated
- Structure prediction gets **combinatorial problem**

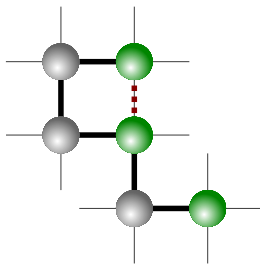
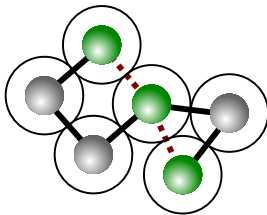
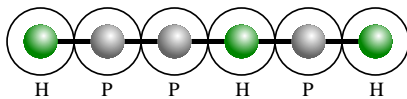
Discrete Structure Space Without Lattice: **Off-lattice models**

- discrete rotational ϕ/ψ -angles of the backbone
- fragment library
- related idea: Tangent Sphere Model

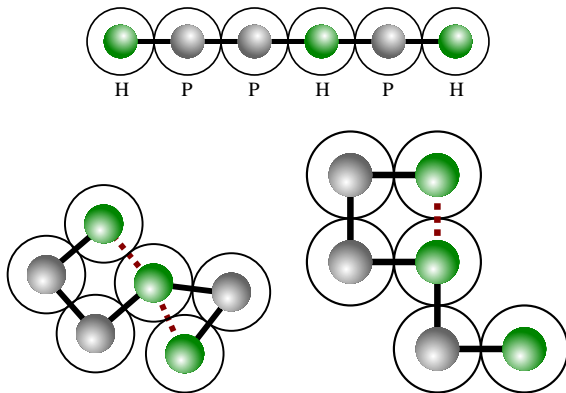
Tangent Sphere Model



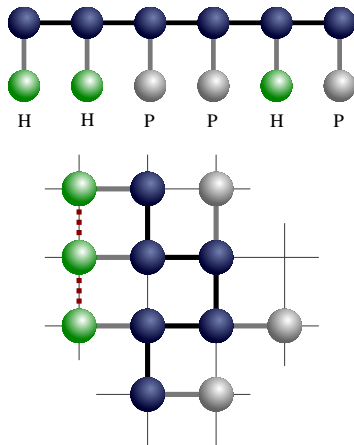
Tangent Sphere Model



Tangent Sphere Model



Side chain models



Lattices

Definition

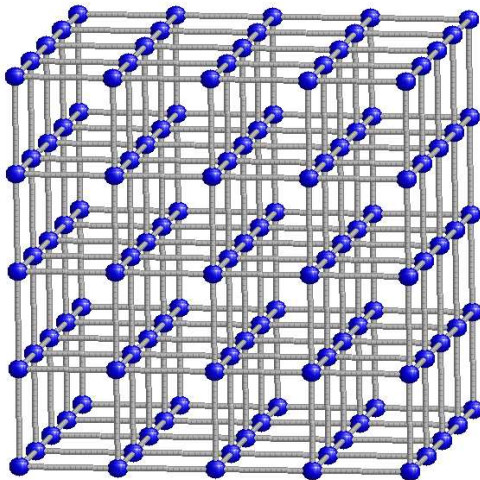
A *lattice* is a set L of *lattice points* such that

$$\vec{0} \in L$$

$$\vec{u}, \vec{v} \in L \text{ implies } \vec{u} + \vec{v}, \vec{u} - \vec{v} \in L$$

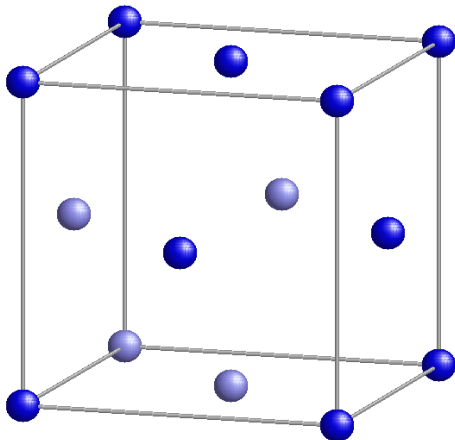
Cubic Lattice

$$\text{Cubic Lattice} = \mathbb{Z}^3$$



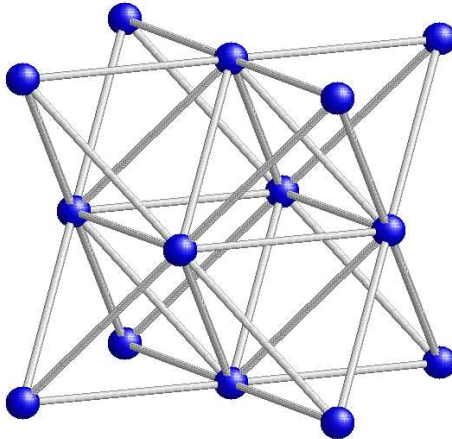
Face-Centered Cubic Lattice (FCC)

$$\text{FCC} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{Z}^3 \mid x + y + z \text{ even} \right\}$$



Face-Centered Cubic Lattice (FCC)

$$\text{FCC} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{Z}^3 \mid x + y + z \text{ even} \right\}$$



The Best Lattice?

- Use protein structures from database PDB
- Generate best approximation on lattice
- Compare off-lattice and on-lattice structure

Measures

$$cRMSD(\omega, \omega') = \sqrt{\frac{1}{n} \sum_{1 \leq i \leq n} \|\omega(i) - \omega'(i)\|^2}$$

$$dRMSD(\omega, \omega') = \sqrt{\frac{1}{n(n-1)/2} \sum_{1 \leq i < j \leq n} (D_{ij} - D'_{ij})^2}$$

$$D_{ij} = \|\omega(i) - \omega(j)\|$$

$$D'_{ij} = \|\omega'(i) - \omega'(j)\|$$

Lattice Approximation - Some Results

Study by Park and Levitt

Lattice	dRMSD	cRMSD
cubic	2.84	2.34
body-centered cubic (BCC)	2.59	2.14
face-centered cubic (FCC)	1.78	1.46

Conclusion

Approximation depends almost only on complexity of the model



Britt H. Park, Michael Levitt. The complexity and accuracy of discrete state models of protein structure Journal of Molecular Biology, 1995

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Lattice/Discrete Models: Pairwise Potentials

- Ab-initio Potentials
 - HP
 - HPNX
(H=Hydrophobic, P=Positive, N=Negative, X=Neutral)
- Statistical Potentials: 20×20 amino acids
 - quasi-chemical approximation (Miyazawa-Jernigan)
 - potential of mean force (Sippl)



Miyazawa S, Jernigan R (1985) Estimation of effective interresidue contact energies from protein crystal structures: quasi-chemical approximation. Macromolecules



Sippl MJ (1990) Calculation of conformational ensembles from potentials of mean force. An approach to the knowledge-based prediction of local structures in globular proteins. J Mol Biol.

Stochastic Local Search

Simulated Annealing & Genetic Algorithms

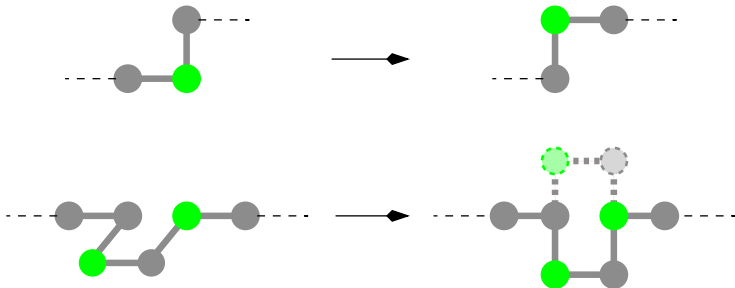


- Applicable to simple or complex protein models
- Heuristic search methods
- Find local optima in energy landscape
- Even for simple models: cannot prove optimality

Move Sets: Local Moves and Pivot Moves

- Stochastic search systematically generates new structures from existing structures
- Idea: new structures are neighbors in the structure space
- New structures generated by applying moves from a *move set*
 - local moves
 - pivot moves

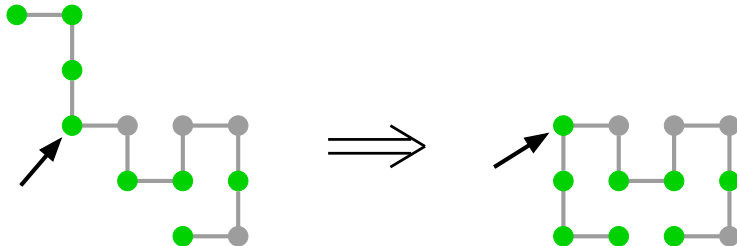
Local Moves



Explanation

A local move changes the positions of a bounded number of monomers at a time.

Pivot Moves



Explanation

A pivot move rotates (and/or reflects) a prefix structure $\omega(1)..\omega(i)$ around $\omega(i)$.

Simulated Annealing — Idea



- Perform a random walk through the structure space by repeatedly applying random moves
- Prefer going to better structures
- Sometimes allow going to worse structures depends on temperature T
 - high T : accept almost all structures
 - low T : accept almost only better structures

Simulated Annealing — Algorithm

Find an optimal structure for sequence s (temperature T)

- Start with random structure ω
- Perform **simulation steps**
 - apply a random **local move** to $\omega \rightarrow \omega'$
 - only accept new structure, i.e. $\omega := \omega'$
 - either if $E(s, \omega') < E(s, \omega)$
 - or with probability

$$\exp\left(-\frac{(E(s, \omega') - E(s, \omega))}{T}\right)$$

- (Cool the temperature down)

Remarks

- Acceptance rule = **Metropolis criterion**
- Guarantee for finding the **global** optimum only for **exponentially slow** cooling. **Otherwise**: we don't know.

(Hybrid) Genetic Algorithm — Idea

- Extend the idea of simulated annealing to **population of structures**
- New structures are generated from existing by
 - Mutation = random pivot move
 - Crossover = random merging two structures

The (Hybrid) Genetic Algorithm [Unger& Moulton]

Find an optimal structure for sequence s

- Generate random start population (e.g. 200 structures)
- Repeat
 - Mutate all structures
 - Generate offspring population by crossover
 - Accept offspring only due to Metropolis criterion
(Here: the energy of each offspring is compared to **average energy** in population.)



R Unger and J Moulton. Local interactions dominate folding in a simple protein model. Journal of Molecular Biology, 1996.

Molecular Dynamics

- Simulates the motion of a protein considering forces between atoms; sounds like the ultimate solution
- Uses **force field** potentials (e.g. AMBER, CHARMM)

$$E_{total} = E_{bonded} + E_{nonbonded}$$

$$E_{bonded} = E_{bond-stretch} + E_{angle-bend} + E_{rotation-along-bond}$$

$$E_{nonbonded} = E_{electrostatic} + E_{van-der-Waals}$$

- Applies Newton's **laws of motion**
- Changes are calculated for **small time steps**
 - small enough to avoid discretization error smaller than vibration of system
 \Rightarrow in order of femtoseconds = 10^{-15} seconds!
 - computationally intensive
 - **critical** for simulation time

Molecular Dynamics: Limits

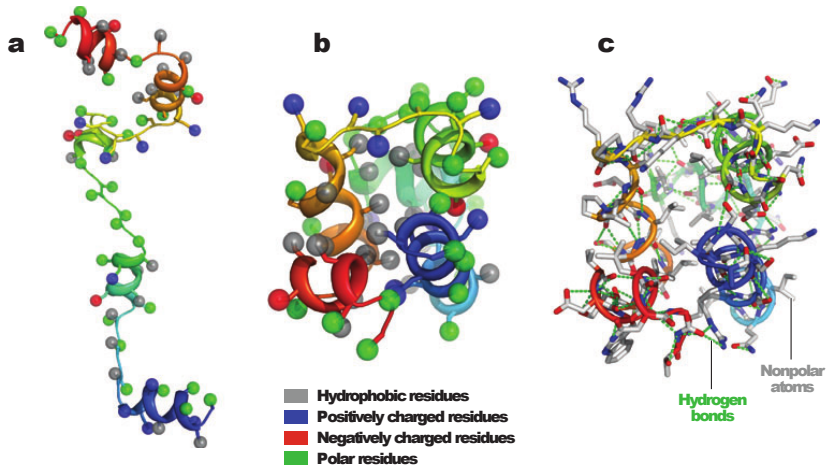
- Simulation gap
Assume one billion steps: $10^{-15} \times 10^9$ is still 10^{-6}
For folding small proteins, we need at least millisecond
- force fields empirical (from comparably small molecules)
valid for protein folding case?
(*"embarrassment of molecular mechanics"*)
- Newton's equations solved numerically (instabilities)
- explicit/implicit solvent
- Quantum MD
- Pair potential/many-body potentials

*Limitations of MD are not exclusively
a matter of computational resources*

Fragment Assembly: Rosetta

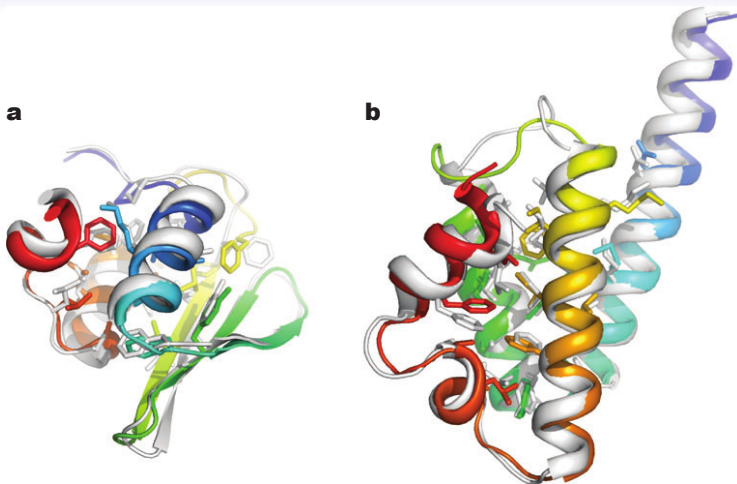
- Monte Carlo search in coarse grained model
- Limit conformational search space by using 9mer motifs
- Rationale
 - Local structures often fold independently of full protein
 - Can predict large areas of protein by matching sequence to motifs
- New structures generated by swapping compatible fragments
- Select candidates for refinement
 - Accepted structures are clustered based on energy and structural size
 - Best cluster is one with the greatest number of conformations within N- rms deviation structure of the center
 - Representative structures taken from each of the best five clusters and returned to the user as predictions

Rosetta: Fragment Assembly and Refinement



Rhiju Das and David Baker. Macromolecular Modeling with Rosetta. Annu. Rev. Biochem, 2008.

Rosetta *de-novo* Blind Prediction Results (CASP6)



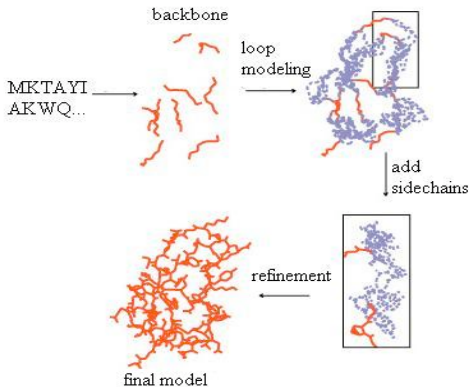
atomic level prediction, $< 2 \text{ \AA}$; a/b: 70/90 residues, 1.6/1.4 \AA

More of Rosetta:



Foldit

Protein Structure Prediction



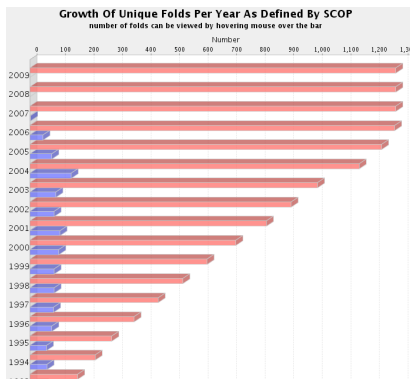
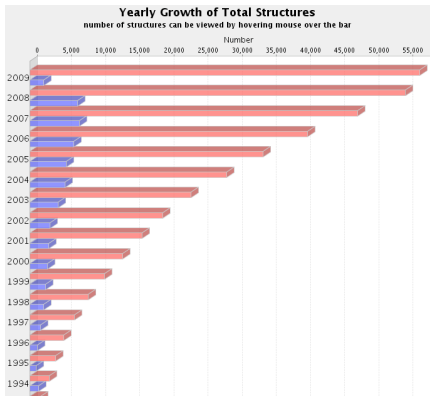
- Stage 1: Backbone Prediction
 - Ab initio folding
 - **Homology modeling**
 - Protein threading
- Stage 2: Loop Modeling
- Stage 3: Side-Chain Packing
- Stage 4: Structure Refinement

The picture is adapted from <http://www.cs.ucdavis.edu/~koehl/ProModel/fillgap.html>

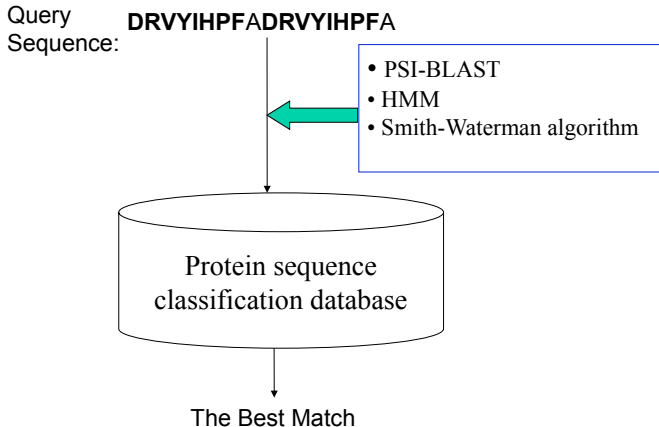
Sometimes grouped “Comparative Modeling”

- **Homology modeling**
 - identification of homologous proteins through sequence alignment
 - structure prediction through placing residues into “corresponding” positions of homologous structure models
- **Protein threading**
 - make structure prediction through identification of “good” sequence-structure fit

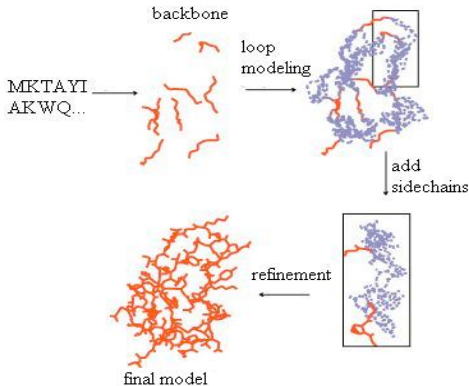
PDB New Fold Growth



Homology Modeling



Protein Structure Prediction



- Stage 1: Backbone Prediction
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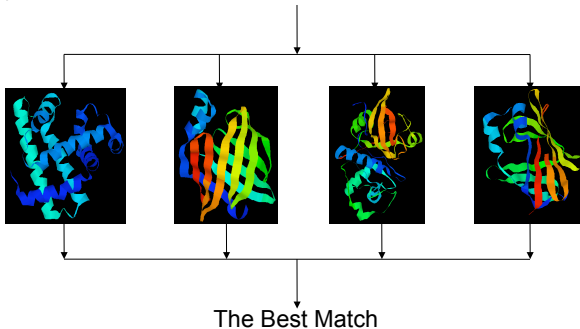
The picture is adapted from <http://www.cs.ucdavis.edu/~koehl/ProModel/fillgap.html>

Protein Threading

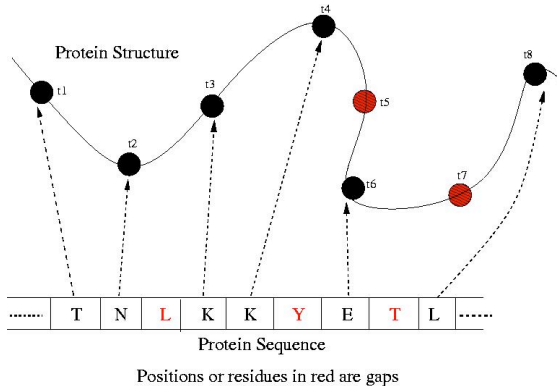
- Make a structure prediction through finding an optimal alignment (placement) of a protein sequence onto each known structure (structural template)
 - “alignment” quality is measured by some statistics-based scoring function
 - best overall “alignment” among all templates may give a structure prediction
- Step 1: Construction of Template Library
- Step 2: Design of Scoring Function
- Step 3: Alignment
- Step 4: Template Selection and Model Construction

Protein Threading

Query Sequence: **DRVYIHPFADRVYIHPFA**



Protein Threading



Threading Model

- Each template is parsed as a chain of cores. Two adjacent cores are connected by a loop. Cores are the most conserved segments in a protein.
- No gap allowed within a core.
- Only the pairwise contact between two core residues are considered because contacts involved with loop residues are not conserved well.
- Global alignment employed

Scoring Function

how preferable to put
two particular
residues nearby: E_p

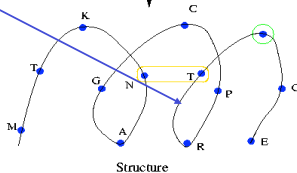
(Pairwise potential)

how well a residue
fits a structural
environment: E_s

(Fitness score)

Sequence: M T K **L I L** N A G C P R T G E **W T Y T E**

threading



alignment gap
penalty: E_g

(gap score)

sequence similarity
between query and
template proteins: E_m

(Mutation score)

How consistent of the secondary structures: E_{ss}

$$E = E_p + E_s + E_m + E_g + E_{ss}$$

Minimize E to find a sequence-template alignment

Scoring: Fitness Score

occurring probability of amino acid a with s

$$\text{FitnessScore}(a, s) = -\log \frac{P(a, s)}{P(a)P(s)}$$

occurring probability of amino acid a

occurring probability of solvent accessibility s

Scoring: Pairwise Potential

occurring probability of a and b with distance < cutoff

$$\textit{PairwisePotential}(a,b) = -\log \frac{P(a,b)}{P(a)P(b)}$$

occurring probability of amino acid a

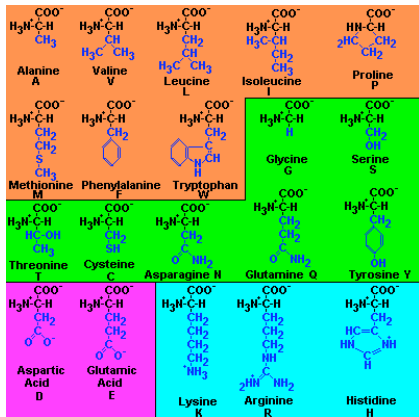
occurring probability of amino acid b

Scoring: Secondary Structure

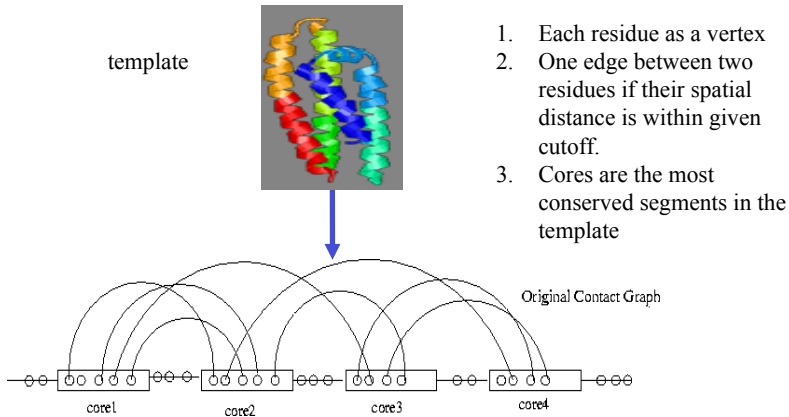
1. Difference between predicted secondary structure and template secondary structure
2. PSIPRED for secondary structure prediction

Scoring: Mutational Score

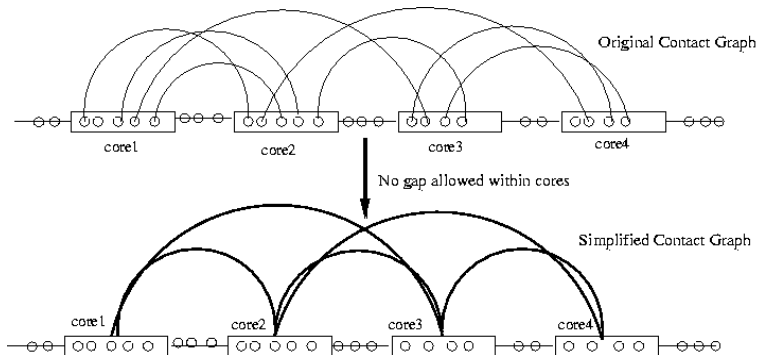
Could be based on chemical similarity, etc, etc.



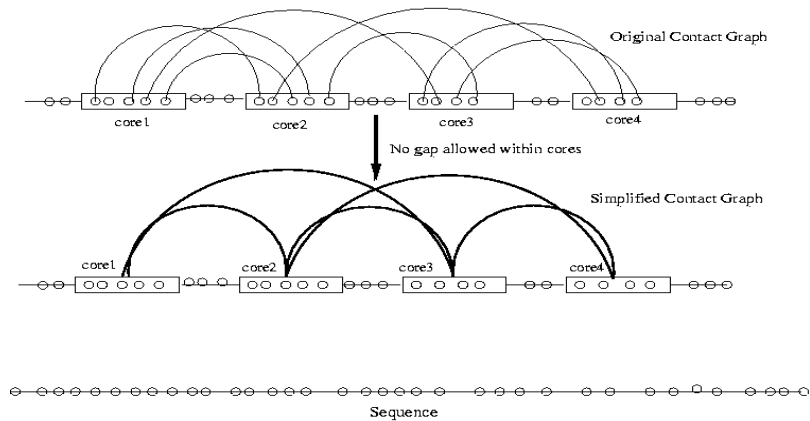
Contact Graph



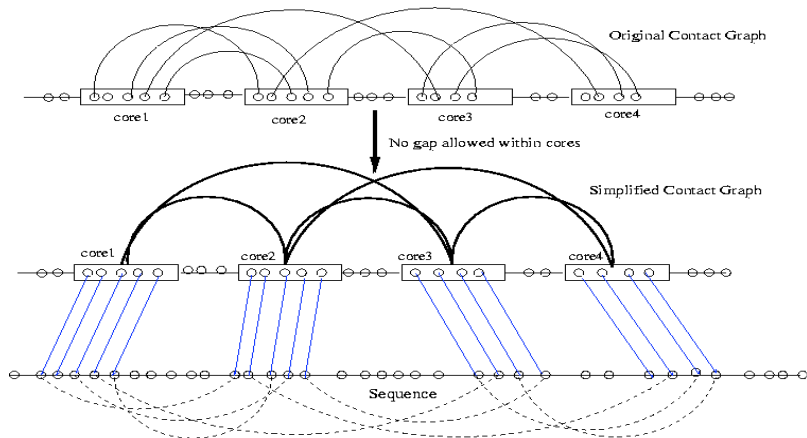
Simplified Contact Graph



Alignment Example



Alignment Example

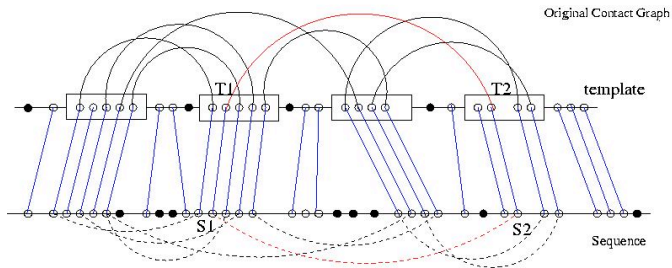


Calculation of Alignment Score

Alignment Score = Singleton Score + Pairwise Score + Gap Penalty

Singleton Score (S1, T1) = Mutation Score(S1, T1) + Fitness Score(S1, T1) + SS(S1, T1)

Pairwise Score = Pairwise Score (S1, S2, dist(T1, T2)) +

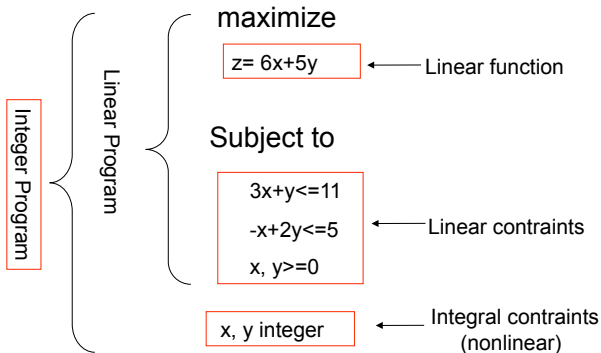


Filled small circles are unaligned template positions or sequence residues

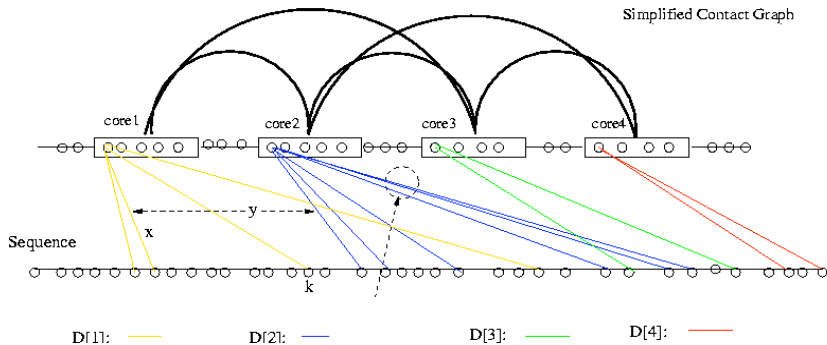
Threading Algorithms

- NP-Hard problem
 - Can be reduced to MAX-CUT
- Approximation Algorithm
 - Interaction-frozen algorithm (A. Godzik et al.)
 - Monte Carlo sampling (S.H. Bryant et al.)
 - Double dynamic programming (D. Jones et al.)
- Exact Algorithm
 - Branch-and-bound (R.H. Lathrop and T.F. Smith)
 - PROSPECT-I uses divide-and-conquer (Y. Xu et al.)
 - Linear programming by RAPTOR (J. Xu et al.)

Linear & Integer Program



Variables



- $x(i,l)$ denotes core i is aligned to sequence position l
- $y(i,l,j,k)$ denotes that core i is aligned to position l and core j is aligned to position k at the same time.

LP Formulation

Minimize

$$E = \sum a_{i,l} x_{i,l} + \sum b_{(i,l)(j,k)} y_{(i,l)(j,k)}$$

a: singleton score parameter

b: pairwise score parameter

s.t.

$$x_{i,l} = \sum_{k \in R[l,j,i]} y_{(i,l)(j,k)}, \forall l \in D[i]$$

$$x_{j,k} = \sum_{l \in R[j,k,i]} y_{(i,l)(j,k)}, \forall k \in D[j]$$

Each y variable is 1 if and only if its two x variable are 1

$$\sum_{l \in D[i]} x_{i,l} = 1$$

Each core has only one alignment position

$$x_{i,l}, y_{(i,l)(j,k)} \in \{0,1\}$$

Online Servers



[http://
www.bioinformatics.uw
aterloo.ca/~j3xu/raptor/
index.php](http://www.bioinformatics.uwaterloo.ca/~j3xu/raptor/index.php)



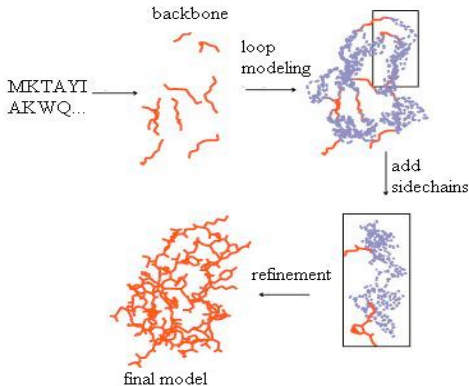
www.bakerlab.org

<http://robetta.bakerlab.org/index.html>

phyre

<http://www.sbg.bio.ic.ac.uk/~phyre/>

Protein Structure Prediction

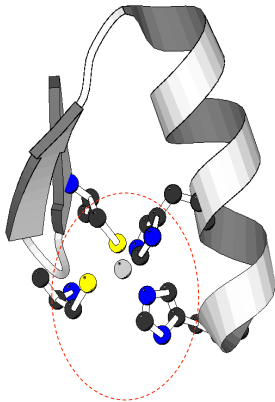


- Stage 1: Backbone Prediction
 - Ab initio folding
 - Homology modeling
 - Protein threading
- Stage 2: Loop Modeling
- Stage 3: Side-Chain Packing
- Stage 4: Structure Refinement

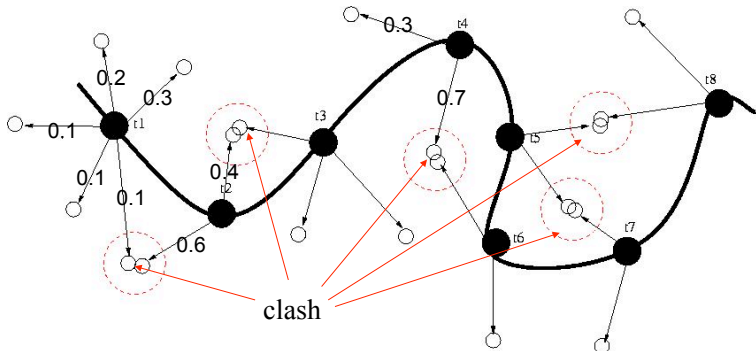
The picture is adapted from <http://www.cs.ucdavis.edu/~koehl/ProModel/fillgap.html>

Protein Side-Chain Packing

- **Problem:** given the backbone coordinates of a protein, predict the coordinates of the side-chain atoms
- **Insight:** a protein structure is a geometric object with special features
- **Method:** decompose a protein structure into some very small blocks



Side-Chain Packing



Each residue has many possible side-chain positions.
Each possible position is called a rotamer.
Need to avoid atomic clashes.

Energy Function

Assume rotamer A(i) is assigned to residue i. The side-chain packing quality is measured by

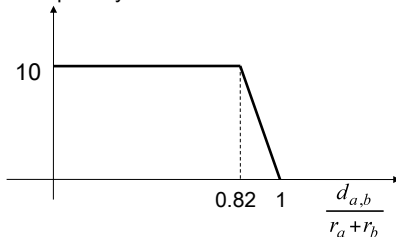
$$\sum_i S(i, A(i)) + \sum P(i, j, A(i), A(j))$$

occurring preference

The higher the occurring probability,
the smaller the value

clash penalty

clash penalty



$d_{a,b}$: distance between two atoms

r_a, r_b : atom radii

Minimize the energy function to obtain the best side-chain packing.

Many Methods

- NP-hard [Akutsu, 1997; Pierce et al., 2002] and NP-complete to achieve an approximation ratio $O(N)$ [Chazelle et al, 2004]
- Dead-End Elimination: eliminate rotamers one-by-one
- SCWRL: biconnected decomposition of a protein structure [Dunbrack et al., 2003]
 - One of the most popular side-chain packing programs
- Linear integer programming [Althaus et al, 2000; Eriksson et al, 2001; Kingsford et al, 2004]
 - The formulation similar to that used in RAPTOR

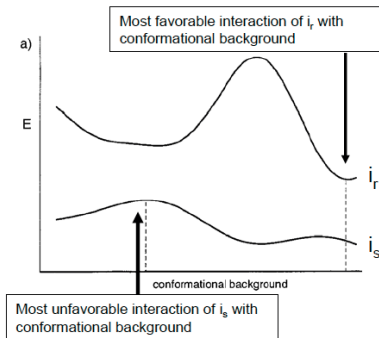
Dead-end elimination

- Conformation consists of N residues, each with a set of r possible rotomers
- Simplification:
Global conformation energy formulated as 2 parts:
 - Sum of all interactions between backbone and N residues
 - Sum of all pairwise interactions between i*j residues
(residues i, j, rotatmers r, s)

$$E_{total} = \sum_{i=1}^N E(i_r) + \sum_{i=1}^{N-1} \sum_{j=i+1}^N E(i_r, j_s)$$

Dead-end elimination

- If two rotamers r, s at residue position i
- can eliminate rotamer s , if pairwise energy between i_r and all other sideschains is **always** higher than pairwise energy between i_s and all other sidechains



Eliminate i_r iff:

$$E(i_r) - E(i_s) + \sum_{j \neq i} \min E(i_r, j) + \sum_{j \neq i} \min E(i_s, j) > 0$$

Dead-end elimination

- Apply iteratively to all rotamer pairs
- After each elimination, energy landscape changes so could cause new elimination that couldn't have happened before