Zuker to Partition Function Variant: Example

$$W_{ij} = \min \begin{cases} W_{ij-1} \\ \min_{i \leq k < j-m} W_{ik-1} + V_{kj} \end{cases}$$

$$Z_{\mathcal{P}_{ij}} = Z_{\mathcal{P}_{ij-1}} + \sum_{i \leq k < j-m} Z_{\mathcal{P}_{ik-1}} \cdot Z_{\mathcal{P}'_{kj}}$$

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Decomposition by Zuker

$$W_{ij} = \min \begin{cases} W_{ij-1} \\ \min_{i \leq k < j-m} W_{ik-1} + V_{kj} \end{cases}$$

$$V_{ij} = \min \begin{cases} \mathrm{eH}(i,j), \min_{i<i'<j'<j} V_{i',j'} + \mathrm{eSBI}(i,j,i',j') \\ \min_{i<k<j} WM_{i+1k-1} + WM_{kj-1} + a \end{cases}$$

$$WM_{ij} = \min \begin{cases} WM_{ij-1} + c, WM_{i+1j} + c, V_{ij} + b \\ \min_{i<k<j} WM_{ik-1} + WM_{kj} \end{cases}$$

Remark

Combine energy function for stacking and interior loop:

$$\mathrm{eSBI}(i,j,i',j') := \begin{cases} \mathrm{eS}(i,j) & i' = i+1 \land j' = j-1 \\ \mathrm{eL}(i,j,i',j') & \text{otherwise} \end{cases}$$

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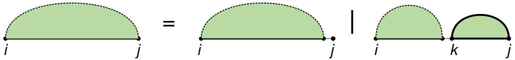

$$V_{ij} = \min \begin{cases} \mathrm{eH}(i,j), \min_{i<i'<j'<j} V_{i',j'} + \mathrm{eSBI}(i,j,i',j') \\ \min_{i<k<j} WM_{i+1k-1} + WM_{kj-1} + a \end{cases}$$

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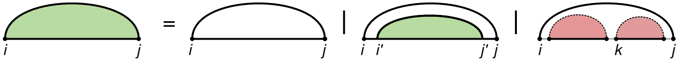

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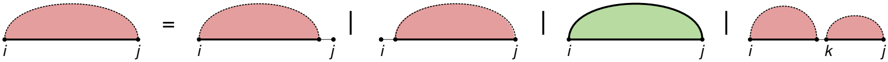

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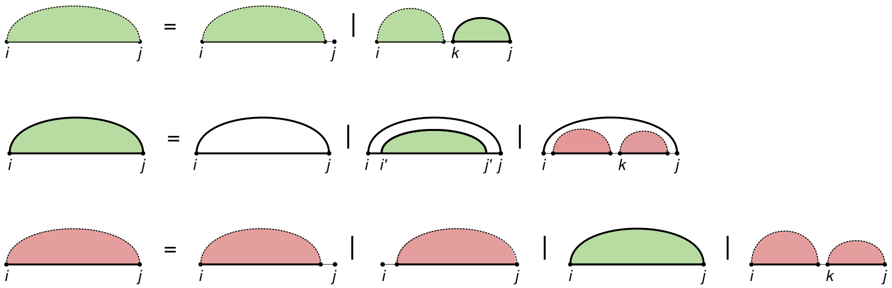
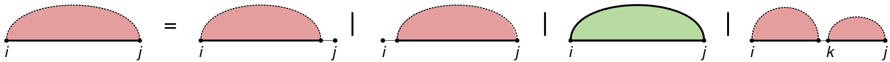
$$\mathrm{eSBI}(i,j,i',j') := \begin{cases} \mathrm{eS}(i,j) & i' = i+1 \land j' = j-1 \\ \mathrm{eL}(i,j,i',j') & \text{otherwise} \end{cases}$$

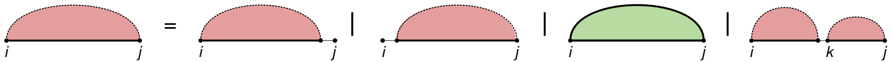
Decomposition of Structure Space by Zuker

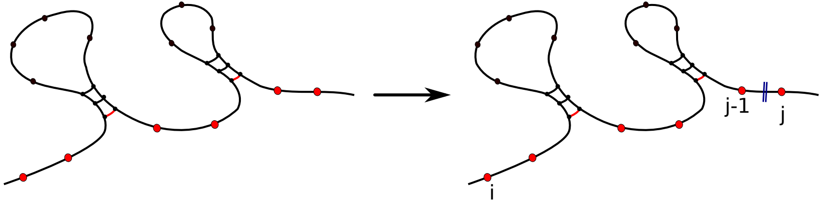


Ambiguity Example



Ambiguity Example





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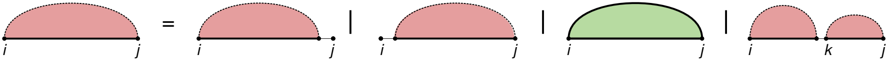
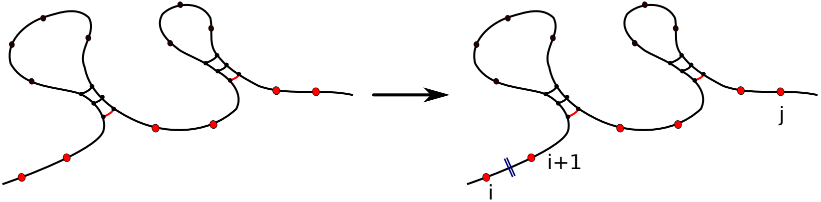
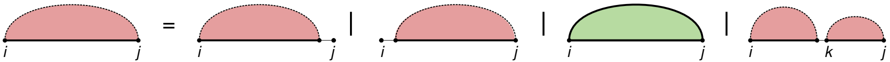
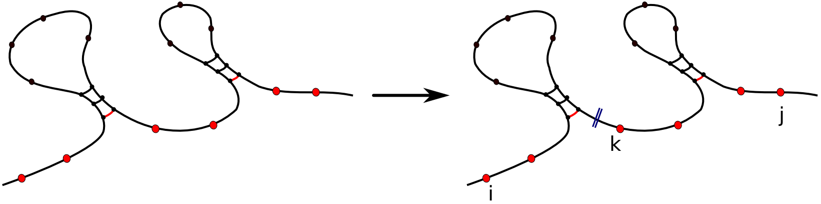


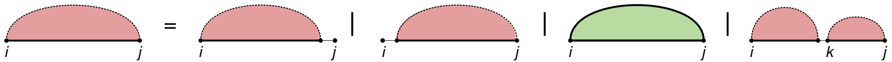


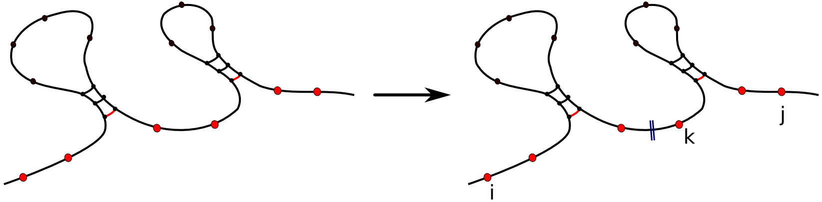
Ambiguity Example



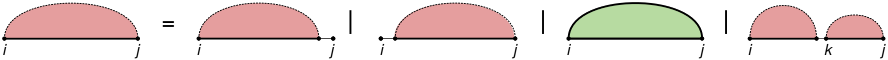


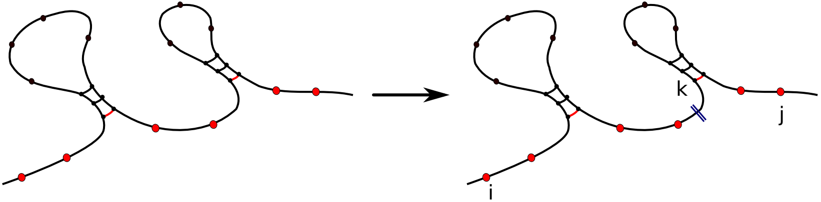
Ambiguity Example





Ambiguity Example





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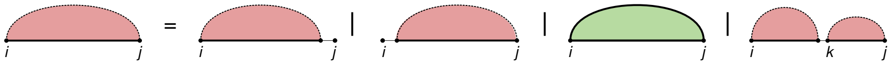
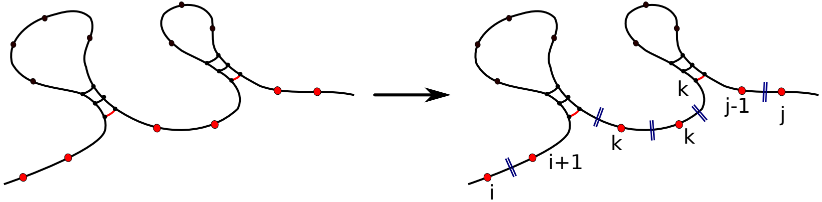
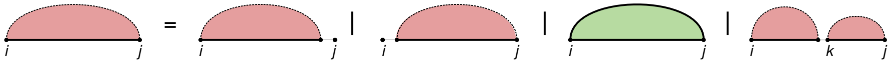
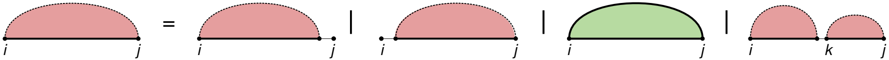


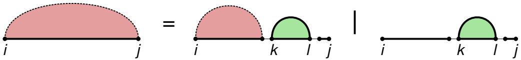


Discarding Ambiguity



Discarding Ambiguity





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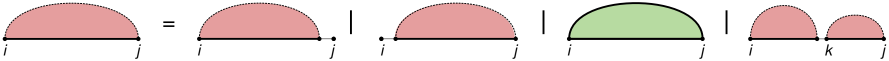

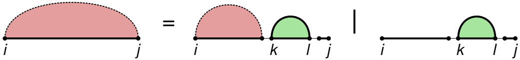

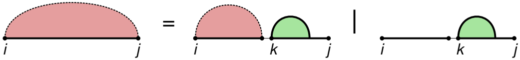

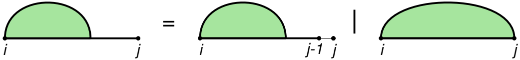

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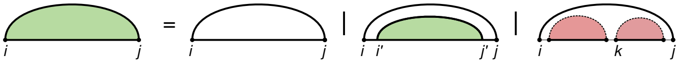

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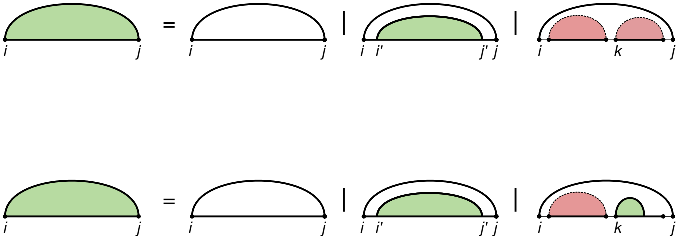
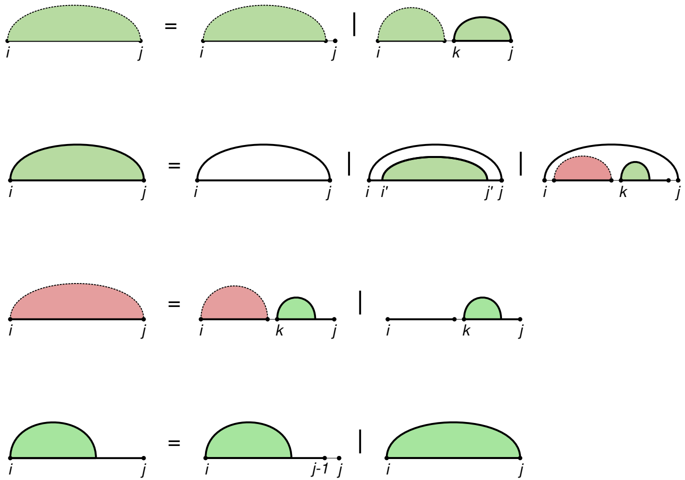


Unambiguous Decomposition: Summary



McCaskill-Algorithm: Matrices

Given RNA sequence $S = S_1 \dots S_n$. As usual, $\mathcal{P}_{ij}$ denotes the ij-subensemble of S.

- Matrix $Q = (Q_{ij})_{1 \leq i < j \leq n}$
- Matrix $Q^b = (Q^b_{ij})_{1 \leq i < j \leq n}$
- Matrix $Q^m = (Q^m_{ij})_{1 \leq i < j \leq n}$
- Matrix $Q^{m1} = (Q^{m1}_{ij})_{1 \leq i < j \leq n}$

McCaskill-Algorithm: Matrices

Given RNA sequence $S = S_1 \dots S_n$. As usual, $\mathcal{P}_{ij}$ denotes the ij-subensemble of S.

- Matrix $Q = (Q_{ij})_{1 \leq i < j \leq n}$

$$Q_{ij} := Z_{\mathcal{P}_{ij}}$$

- Matrix $Q^b = (Q^b_{ij})_{1 \leq i < j \leq n}$

$$Q^b_{ij} := Z_{\mathcal{P}^b_{ij}}, \text{ where } \mathcal{P}^b_{ij} := \{P \in \mathcal{P}_{ij} \mid (i,j) \in P\}$$

- Matrix $Q^m = (Q^m_{ij})_{1 \leq i < j \leq n}$

$$Q^m_{ij} := Z^m_{\mathcal{P}^m_{ij}}, \text{ where } \mathcal{P}^m_{ij} := \{P \in \mathcal{P}_{ij} \mid P \text{ non-empty}\},$$

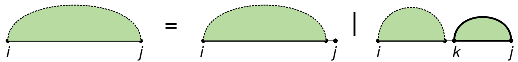
  Z^m includes multiloop-contributions (for unpaired bases, inner base pairs), see earlier modification E^m.

- Matrix $Q^{m1} = (Q^{m1}_{ij})_{1 \leq i < j \leq n}$

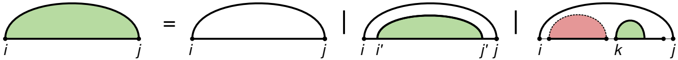
$$Q^{m1}_{ij} := Z^m_{\mathcal{P}^{m1}_{ij}},$$

  where $\mathcal{P}^{m1}_{ij} := \{P \in \mathcal{P}_{ij} \mid \exists k : (i,k) \in P \wedge k+1, \dots, j \text{ unpaired in } P\}$.

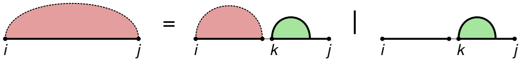
Q-recursion



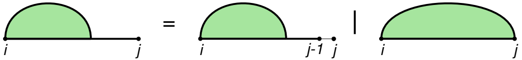
Q^b-recursion



Q^m-recursion



Q^{m1}-recursion



McCaskill — Partition Function — Summary

$$Q_{ij} = 1 \quad (i \geq j - m)$$

$$Q_{ij} = Q_{ij-1} + \sum_{i \leq k < j-m} Q_{ik-1} \cdot Q^b_{kj}$$

$$Q^b_{ij} = 0 \quad (i \geq j - m)$$

$$Q^b_{ij} = \exp(-\beta\, \mathrm{eH}(i,j)) + \sum_{i<i'<j'<j} \exp(-\beta\, \mathrm{eSBI}(i,j,i',j')) \cdot Q^b_{i'j'} + \sum_{i<k<j-m-1} Q^m_{i+1k-1} \cdot Q^{m1}_{kj-1} \cdot \exp(-\beta a)$$

$$Q^m_{ij} = 0 \quad (i \geq j - m)$$

$$Q^m_{ij} = \sum_{i \leq k < j-m} (\exp(-\beta(k-i)c) + Q^m_{ik-1}) \cdot Q^{m1}_{kj}$$

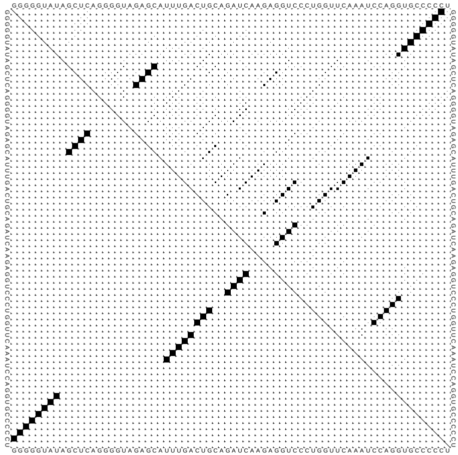
$$Q^{m1}_{ij} = 0 \quad (i \geq j - m)$$

$$Q^{m1}_{ij} = \sum_{i+m<k\leq j} Q^b_{ik} \cdot \exp(-\beta b) \cdot \exp(-\beta(j-k)c)$$

McCaskill Remarks

- Partition function of the ensemble of S in $Z = Q_{1n}$
- Correctness due to disjoint (=unambiguous) and independent decomposition
- Complexity $O(n^2)$ space,
  $O(n^3)$ time (after bounding size of interior loops)
- Probabilities
  - of a structure
    $\Pr[P|S] = Z^{-1} \exp(-\beta E(P))$ (efficient!)
  - of a structure set
    $\Pr[\mathcal{P}|S] = Z^{-1} \sum_{P \in \mathcal{P}} \exp(-\beta E(P))$ (depends on $|\mathcal{P}|$)
  - of a base pair (i, j)
    $\Pr[(i,j)|S] = Z^{-1} \sum_{P \ni (i,j)} \exp(-\beta E(P))$ (?)

Base Pair Probabilities



$$\Pr[(i,j)|S] := \sum_{P \ni (i,j)} \Pr[P|S]$$