18.395 Group Theory with Applications to Physics

Fall Term, 2012 | Prof. D. Freedman
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Prerequisites : Students should have a good knowledge of linear algebra, i.e., vector space theory and matrices. They should have completed a course in quantum mechanics in which the theory of angular momentum was discussed and which (hopefully but not essentially) included a discussion of representations of the angular momentum algebra.

Course Content : The aspects of group theory discussed in this course have been selected because of their usefulness in physics. However there will probably be limited time for discussion of physics motivations or applications. In the past most students in the course have been graduate students or advanced undergraduates in applied mathematics and physics. The general philosophy is to use finite groups to bring out some major ideas in a simple context, but to emphasize Lie groups, Lie algebras, and their representations. The outline below gives an "approximate" list of topics to be treated.

Part I: General Group Theory (30%)

1. Definition of a group — abelian or non-abelian — $GL(n, R)$, $GL(n, C)$, and their subgroups — Z_n and the group of the triangle.

2. Rotation group $O(3)$ — proper and improper orthogonal matrices. Rotations about the coordinate axes and their generators. Euler's theorem — Euler angles from the viewpoint of the stability subgroup.

3. Lorentz group — rotations, boosts, and their generators.

4. Group homomorphisms—relations between $SL(2, C)$ and the proper Lorentz group — $SU(2)$ and $SO(3)$.

5. Matrix Lie groups and matrix Lie algebras.

6. Cosets and Conjugacy Classes.

Part II: Representations of Groups (15%)

1. Definitions and terminology – two viewpoints, matrices and linear transformations on carrier spaces — unitary reps. — invariant subspaces, reducible and irreducible reps.

2. Direct sums and Direct products.

3. Schur's Lemmas.

4. Statement (with few proofs) of the basic theorems on the representations of finite groups.

Part III: Dirac Matrices and Representations of Clifford Algebras (10%)
This is a major application of the material of part II.

Part IV: Lie Algebras and their Representations (45%)

1. Definitions — basis — structure constants — Cartan-Killing form — simple and semi-simple — compact and non-compact.

2. Representations of Lie algebras — the adjoint rep — review of $su(2)$.

3. Cartan-Weyl-Dynkin approach to simple Lie algebras and their irreps — Cartan subalgebra and roots — weight and root vectors and their geometrical relations — the **Master Formula** — root and weight diagrams for $su(2)$ and $su(3)$.

4. Positive and highest weights — simple roots and their geometry — simple roots determine all roots — Dynkin diagrams and the Cartan matrix.

5. C-W-D approach to irreps of compact simple Lie algebras. Examples $su(3)$, $g(2)$, $so(2n)$.

Work: Students are expected to complete and turn in problem sets which will be assigned approximately biweekly. The total number of problems is about 25. Grades will be based on the problem sets. There is no final exam.