

Name _____

18.311, Principles of Applied Mathematics, Spring 2006, Prof. Bazant

Midterm Exam – Monday, April 10, 2006

Instructions: Please write your name on every page. This closed-book exam will last 55 minutes. Each problem has roughly equal grading weight.

1. Solve for the car density $\rho(x, t)$ with initial condition,

$$\rho(x, 0) = \begin{cases} 0 & \text{for } x < -d \\ \rho_j/2 & \text{for } |x| < d \\ \rho_j & \text{for } x > d \end{cases}$$

for the linear flow-density relation, $q(\rho) = u_m \rho(1 - \rho/\rho_j)$. Sketch characteristics in space-time, and label any expansion fans or shocks.

2. Solve for the river area profile $A(x, t)$ satisfying

$$A_t + \alpha A^{1/4} A_x = 0 \quad \text{for } x > 0, t > 0$$

$$A(x, 0) = A_0 \quad \text{and} \quad A(0, t) = 0$$

after a dam interrupts the flow. (What are the units of α ?) Sketch characteristics in space-time, and label any expansion fan or shock.

3. Solve

$$\sqrt{u} u_x + u_y = -u$$

subject to $u = x^2$ for $y = 0, x > 0$.