Lecture 19.

$$A = A(x,t,u)$$
$$F = F(x,t,u)$$

$$\underbrace{u_t + A u_x = F}$$

A real diagonalizable

$$A\vec{r}_n = \lambda_n \vec{r}_n$$
$$\vec{l}_n A = \lambda_n \vec{l}_n \qquad \vec{l}_n \cdot \vec{r}_m = \delta_{nm}$$

$$\boxed{\vec{l}_n \cdot (u_t + \lambda_n u_x) = \vec{l}_n \cdot F}$$ ch. form

$$\frac{dx}{dt} = \lambda_n \qquad \vec{l}_n \cdot \frac{dx}{dt} = \vec{l}_n \cdot F$$

Last lecture looked at $A = \text{const}$

Example: Linear first : $A = A(x,t)$, $F = F(x,t)$

Sound waves

$$\boxed{\begin{aligned} E &= \tfrac{1}{2}u^2 + e \\ e &= e(T) \end{aligned}}$$

$$\boxed{\begin{aligned} &\rho_t + (\rho u)_x = 0 \\ &(\rho u)_t + (\rho u^2 + p)_x = 0 \\ &(\rho E)_t + (\rho u E + p u)_x = 0 \end{aligned}}$$

Equ. of state

$$\boxed{e = e(p,\rho)}$$

$$\boxed{\begin{aligned} &\rho_t + (\rho u)_x = 0 \\ &\rho u_t + \rho u u_x + p_x = 0 \\ &\rho E_t + \rho u E_x + (pu)_x = 0 \end{aligned}}$$

$$\rho(\cancel{u}u_t + e_t) + \rho(\cancel{u}u_x + e_x) + \cancel{u p_x} + p u_x = 0$$

$$\rho e_t + \rho u e_x + p u_x = 0$$

$$\rho_t + \rho u_x + \rho_x u = 0 \qquad u_x = -\frac{1}{\rho}\rho_t - \frac{1}{\rho} u \rho_x$$

$$\underbrace{\rho e_t + \rho u e_x - \frac{1}{\rho}[\rho_t + u\rho_x] p = 0}$$

$$\boxed{v = \frac{1}{\rho}} \qquad \underbrace{v_t + u v_x = -\frac{1}{\rho^2}(\rho_t + u\rho_x)}$$

$$\boxed{\cancel{\rho}(e_t + u e_x) + \cancel{\rho} p (v_t + u v_x) = 0}$$

$$\boxed{p\,dv + de = T\,dS}$$

In this case $p\,dv + de$ is differential with 2 unknowns ∴ it has integrating factor

p-set #2

Generic case

$$\boxed{p\,dv + de + \sum \lambda_j d\phi_j = T\,dS}$$

$\boxed{e = e(p, e)}$

Intro. to wave propagation in nonlinear fluids and solids
Drumheller

$$p\frac{dv}{dt} + \frac{de}{dt} = T\frac{dS}{dt} \implies$$

3rd eqn becomes $\frac{dS}{dt} = 0$

System wow

Lab frame

$$\rho_t + (\rho u)_x = 0$$
$$u_t + u u_x + \frac{1}{\rho} p_x = 0$$
$$S_t + u S_x = 0$$

particle path

$$\boxed{\frac{Dx}{Dt} = u}$$

$$\frac{D\rho}{dt} + \rho u_x = 0 \qquad V = \frac{1}{\rho}$$
$$\frac{Du}{Dt} + V p_x = 0$$
$$\frac{DS}{Dt} = 0$$

Characteristic Form Particle path

$$\boxed{p = p(\rho, S)}$$

$$\left(\frac{\partial p}{\partial S}\right)_\rho = \sigma$$

$$\left(\frac{\partial p}{\partial \rho}\right)_S = c^2$$

$$\frac{D\rho}{Dt} + \rho u_x = 0$$
$$\frac{Du}{Dt} + V\left[c^2 \rho_x + \sigma S_x\right] = 0$$
$$\boxed{\frac{DS}{Dt} = 0}$$

multiply top by $\frac{c}{\rho}$ and add to middle

$$\boxed{\frac{Du}{Dt} + c u_x} + \frac{c}{\rho}\left(\frac{D\rho}{Dt} + c\rho_x\right) + v\sigma S_x + \frac{v\sigma}{u}\frac{DS}{Dt} = 0$$

$$\boxed{u_t + (u+c)u_x}$$

$$\boxed{\frac{dx}{dt} = u + c}$$

$$\boxed{\frac{du}{dt} + \frac{c}{\rho}\frac{d\rho}{dt} + \frac{v\sigma}{u}\frac{ds}{dt} = 0}$$

Equation

$$\rho u\frac{du}{dt} + cu\frac{d\rho}{dt} + \sigma\frac{ds}{dt} = 0.$$

on $\frac{dx}{dt} = u + c$

$c \to -c$

Gas dynamics ch. one

c = speed of sound

$\frac{dx}{dt} = u$ ← particle paths

$$\boxed{\frac{dx}{dt} = u \pm c}$$

Sound waves

$$Y_t + AY_x = 0$$

$$Y = \begin{bmatrix} \rho \\ u \\ s \end{bmatrix}$$

$$\rho_t + \rho u_x + u\rho_x = 0$$

$$u_t + uu_x + \frac{1}{\rho}p_x = 0$$

$$S_t + uS_x = 0$$

$$A = \begin{bmatrix} u & \rho & 0 \\ \frac{1}{\rho}c^2 & u & \frac{1}{\rho}\sigma \\ 0 & 0 & u \end{bmatrix}$$

$$\det(\lambda - A) = (\lambda - u)\left[(\lambda-u)^2 - c^2\right] = 0$$

slow $\lambda = u$ & $\lambda = u \pm c$

$$Y = \begin{bmatrix} \rho \\ \rho u \\ \rho E \end{bmatrix} \qquad Y_t + A Y_x = 0$$

Constant entropy

Infinitesimal perturbation)

$$\boxed{p = p(\rho)}$$

$$\rho \frac{du}{dt} + c\frac{dp}{dt} = 0$$

$$\rho \frac{du}{dt} - c\frac{dp}{dt} = 0$$

$$\boxed{\frac{dx}{dt} = u \pm c}$$

$\rho \approx \rho_0$

$c \approx c_0$

$u \approx u_0$

$$\rho_0 \frac{du}{dt} \pm c_0 \frac{dp}{dt} = 0$$

$$\frac{dx}{dt} = u_0 \pm c_0$$

$\rho_0 u + c_0 p$ const. along $\frac{dx}{dt} = u_0 + c_0$

$\rho_0 u - c_0 p$ " " $\frac{dx}{dt} = u_0 - c_0$

$$\boxed{\begin{aligned} \rho_0 u + c_0 p &= f(x - (u_0 + c_0)t) \\ \rho_0 u - c_0 p &= g(x - (u_0 - c_0)t) \end{aligned}}$$

Acoustics