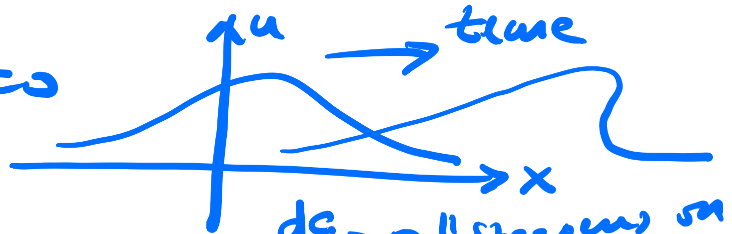
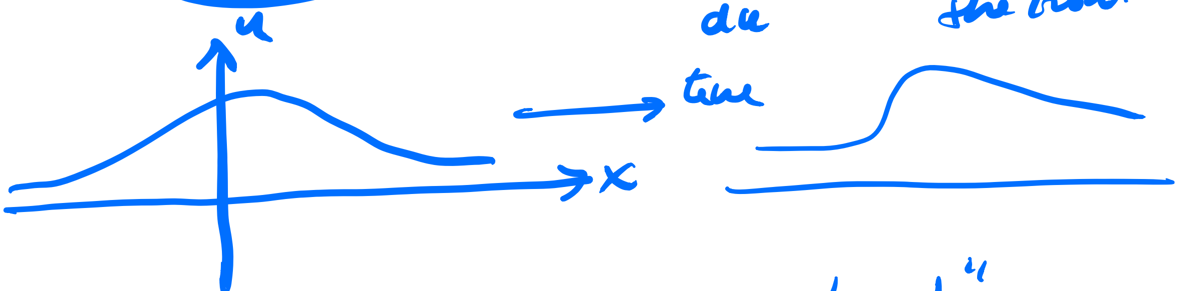
Lecture 11 Resolution of Multiple Values

$u_t + c(u)u_x = 0$

$\frac{dc}{du} \neq 0$



$\frac{dc}{du} > 0$ "steepens on the front"

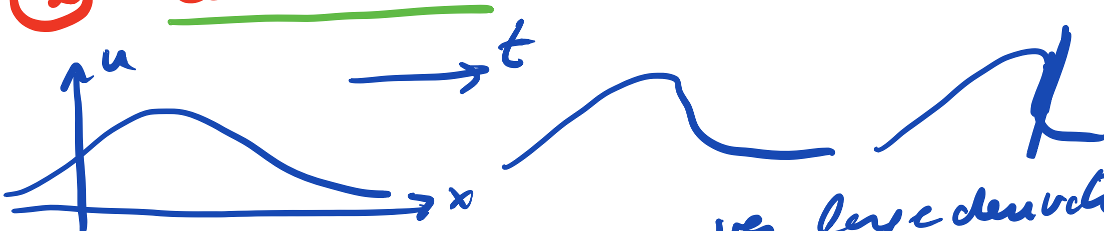


$\frac{dc}{du} < 0$ "steepens on the back"

Where is the failure in the modeling?

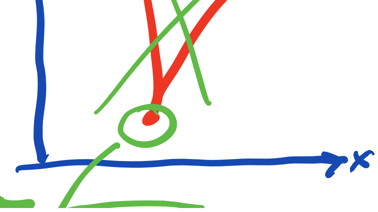
1. Cont. approx.
2. Conservation
3. Quasi-equil.

Fails



very large derivative

Right before multiple values $\frac{\partial u}{\partial x} \to \infty$



Quasi-equilibrium breaks

down $\quad u_x = u_t = \infty$

Approaches

a) Kitchen sink model everything!

b) Is there a simpler way?

For a large class of problems (physical problems) this is true: "dissipation dominated" problems.

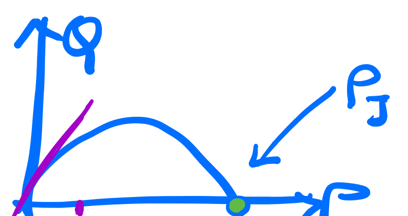
Example (aiming to "b")

Traffic Flow

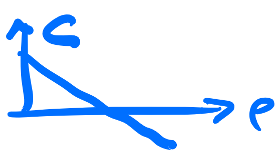
Example #1 $\boxed{\text{Red light turns green}}$

$$\rho_t + Q(\rho)_x = 0 \longleftrightarrow \rho_t + c(\rho)\rho_x = 0$$

$$c = \frac{dQ}{d\rho}$$

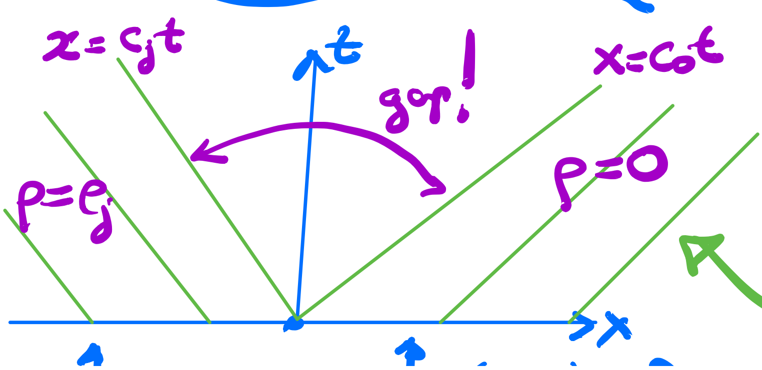


$\frac{dc}{d\rho} < 0$



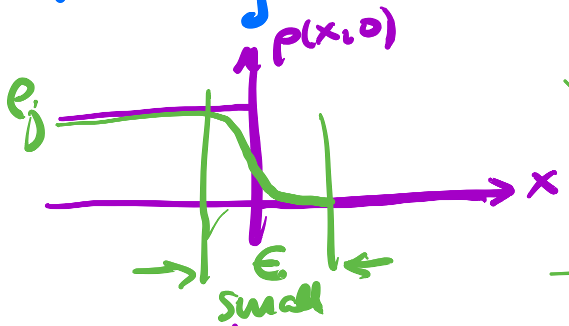
Traffic light at $x=0$

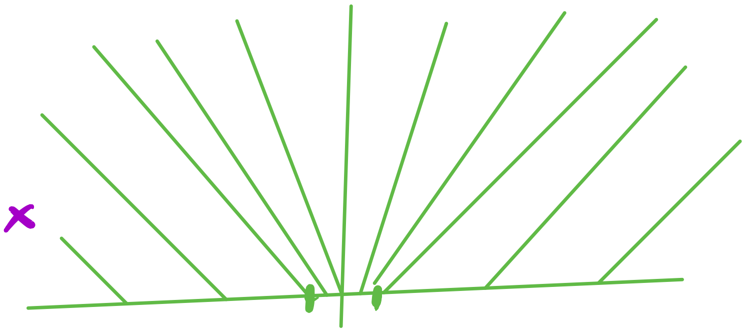
Turns green at $t=0$

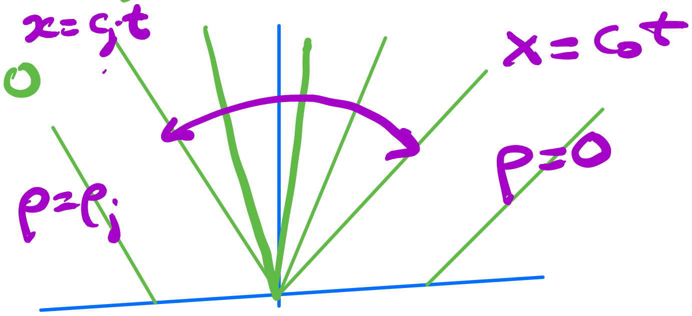


slope $c_0 > 0$

$\rho(x,0) = \rho_j$ $\qquad$ $\rho(x,0) = 0$







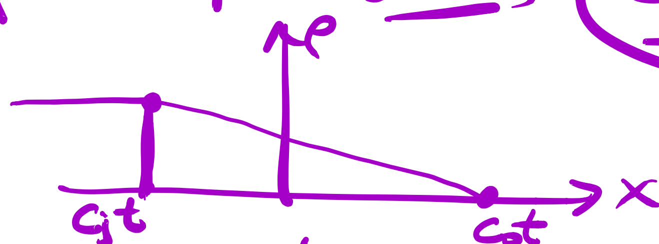
$x = \tilde{c}\, t$

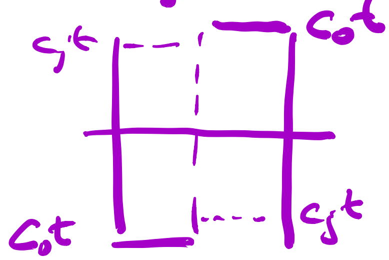
$c_j < \tilde{c} < c_0$

$c(\rho) = \tilde{c}$

ρ follows from solving $c(\rho) = \frac{x}{t}$

$t > 0$

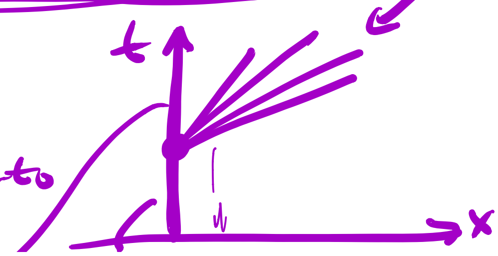




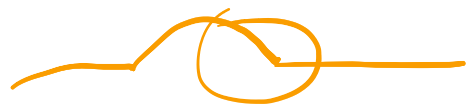
1^{st} con in line

Expansion Fans

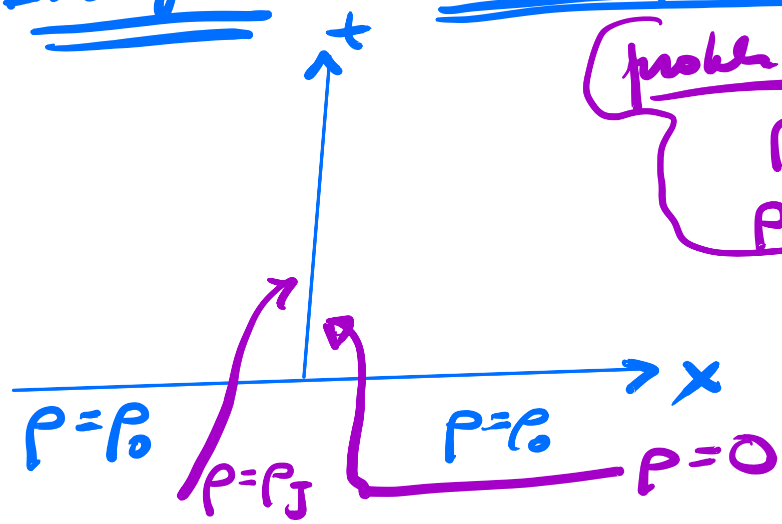
$x - x_0 = c(\rho)(t - t_0)$



$\rho=\rho_2 \quad \rho=\rho_1 \quad c(\rho_2) < c(\rho_1)$



Example 2 Green light turns red

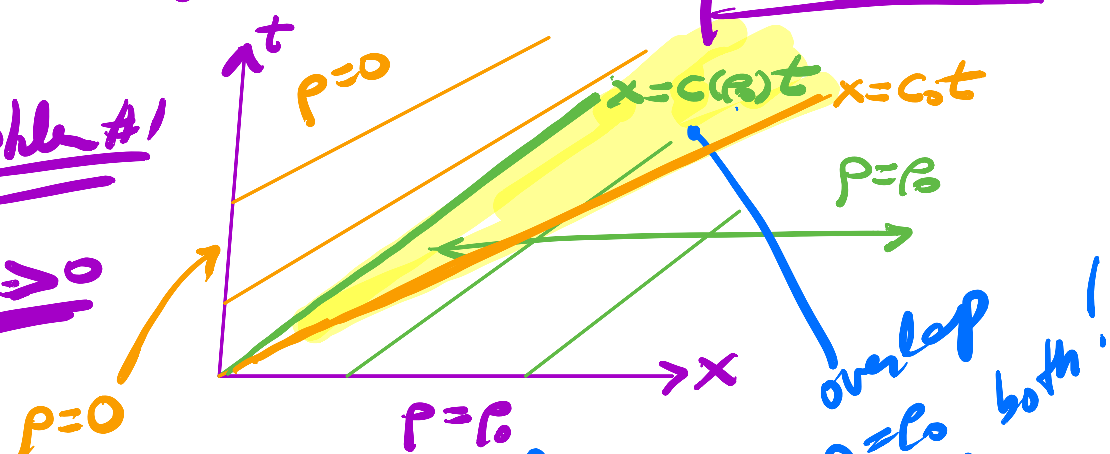


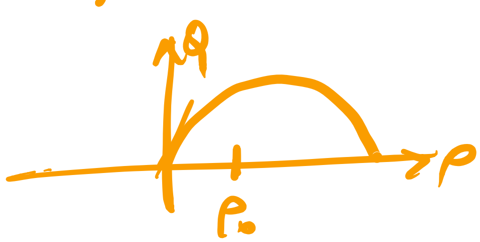
problem #1 $x>0,\ t>0$
$\rho=\rho_0$ for $t=0,\ x>0$
$\rho=0$ for $t>0$ $x=0$

prm #2 $x<0,\ t>0$
$\rho=\rho_0$ for $t=0,\ x>0$
$\rho=\rho_j$ for $x=0,\ t>0$

Problem #1

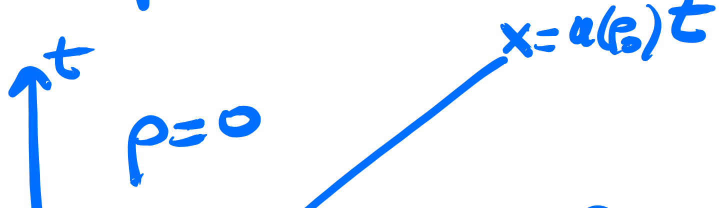
$c(\rho_0) > 0$

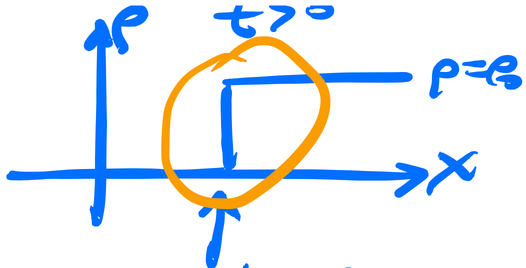




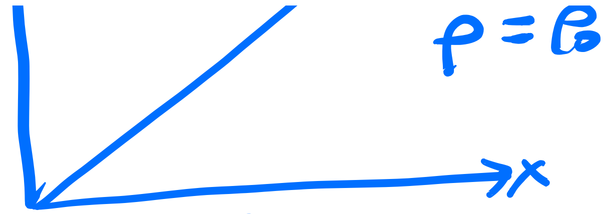
"Maths"

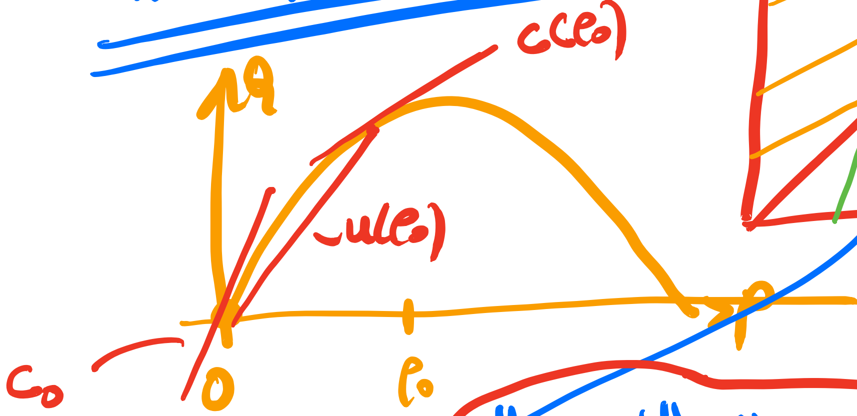
Actuality

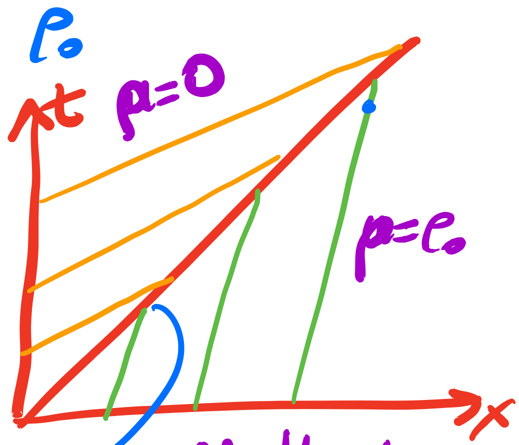




last car through light



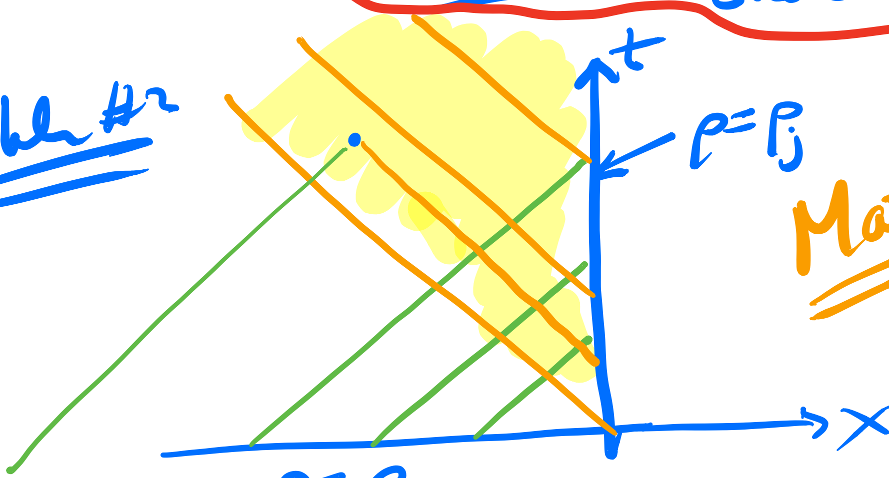




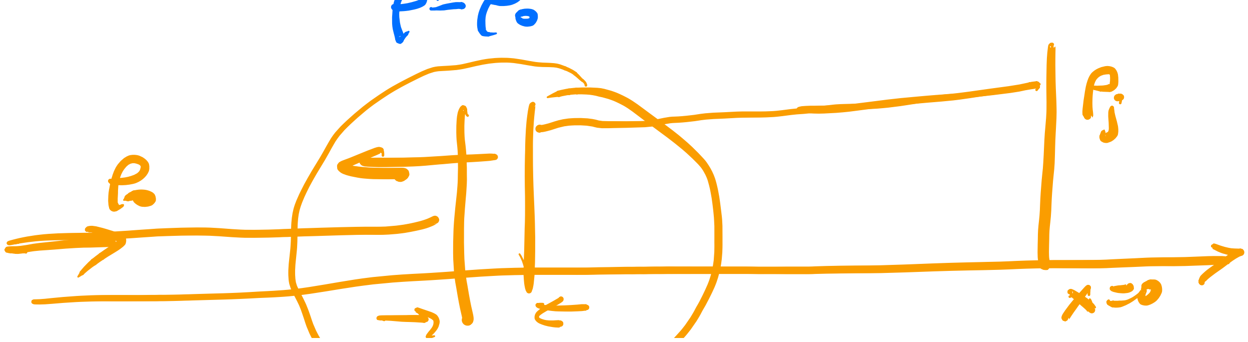
Math + physics

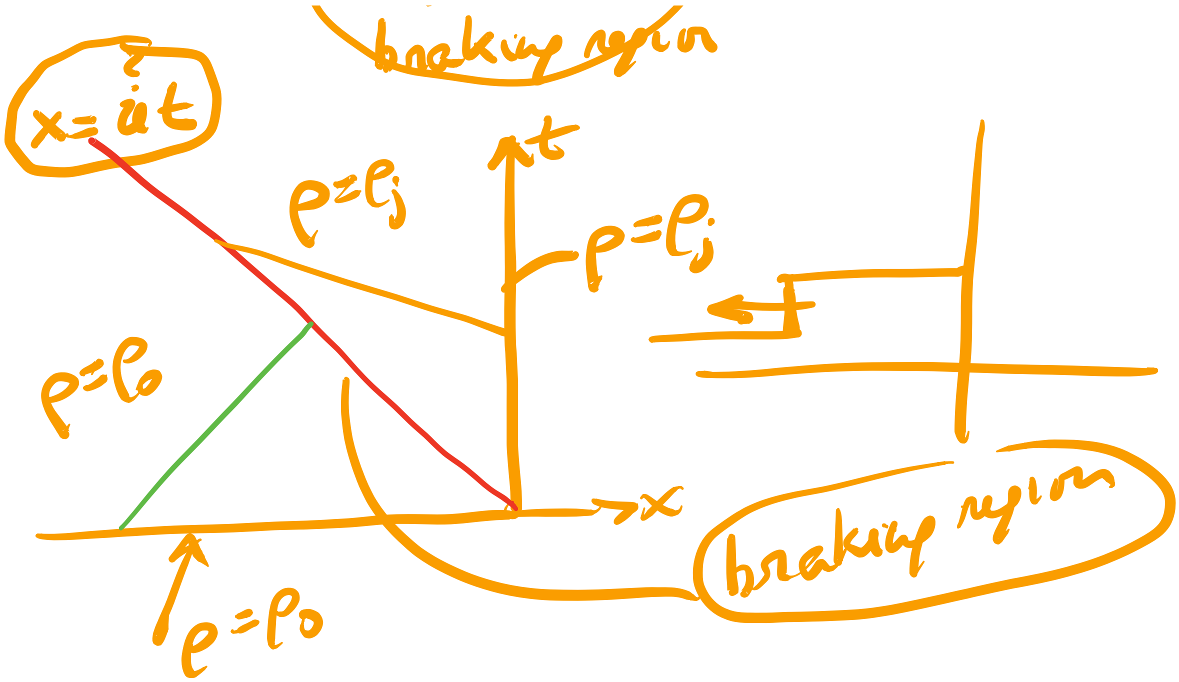
"Shock" = "cemetery of charact."

Problem #2

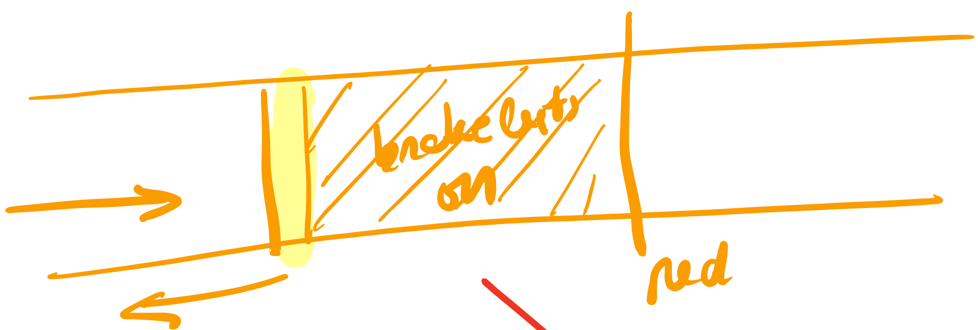


Math

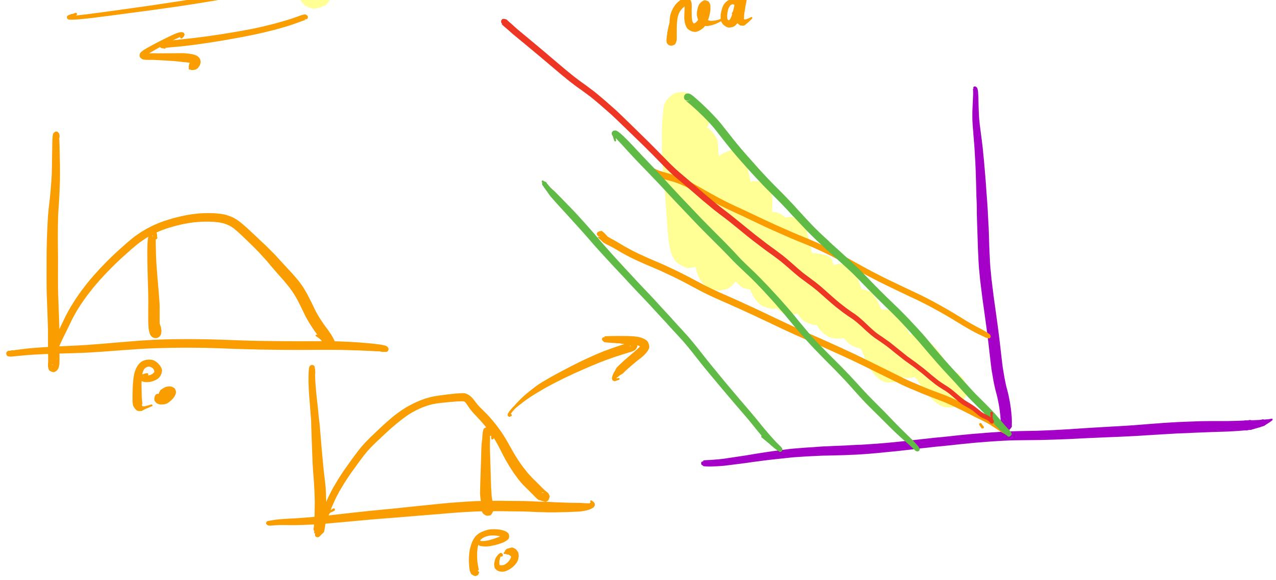




braking region
$x = \dot{u}t$
$\rho = \rho_j$
$\rho = \rho_j$
t
x
$\rho = \rho_0$
$\rho = \rho_0$
braking region



brake lights on
red



ρ_0
ρ_0