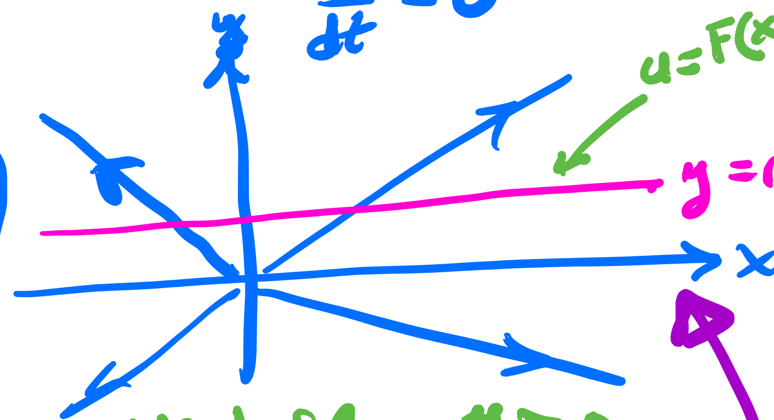
Example:

$$\boxed{x u_x + y u_y = 0} \qquad \boxed{\frac{dx}{dt} = x, \quad \frac{dy}{dt} = y}$$

$$\frac{du}{dt} = 0$$

$$\boxed{\begin{aligned} x &= c_1 e^t \\ y &= c_2 e^t \\ u &= c_3 \end{aligned}}$$



Want soln $y > 0$

Alternative

$$x u_x + y u_y = 0$$

Ch. $\boxed{\frac{dx}{dy} = \frac{x}{y}}$

$y > 0$ $\quad \frac{x}{y} u_x + u_y = 0$ $\quad \boxed{x = c y}$

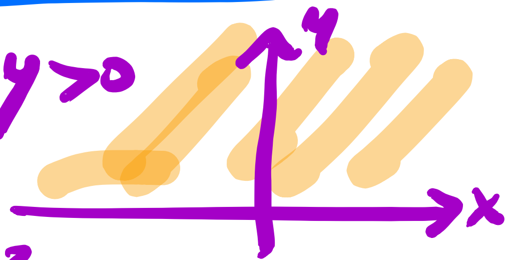
$c = \frac{x}{y}$ $\quad \frac{du}{dy} = 0$ $\quad u = f(c)$

$$\boxed{u = f\left(\frac{x}{y}\right)}$$

Sln problem where $u(x, 1) = F(x)$

is $\boxed{u = F(x/y)}$

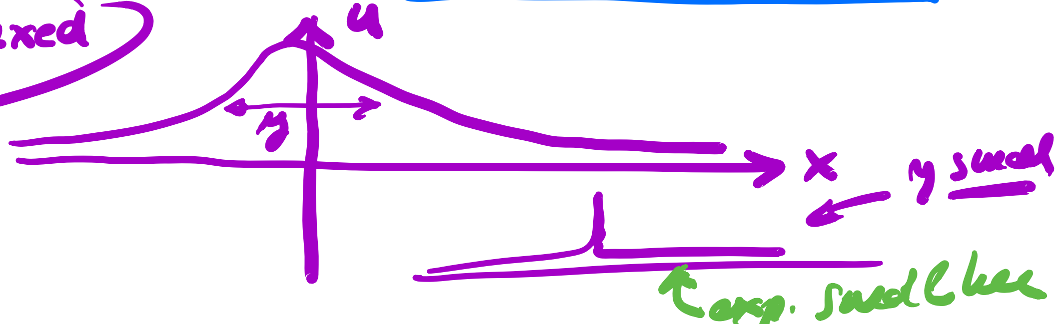
Defined only on $y>0$



Example $F(x) = e^{-x^2}$

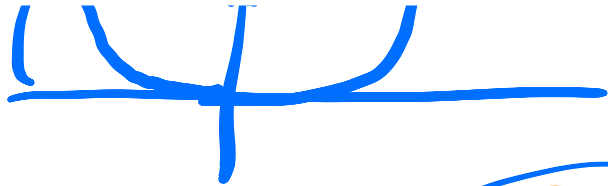
$$\Longrightarrow \quad u = e^{-(x/y)^2}$$

y fixed



Note e^{-x^2/y^2} is smooth function **everywhere** but $x=y=0$

$$u = \begin{cases} e^{-x^2/y^2} & \text{for } y>0 \\ c\,e^{-x^2/y^2} & \text{for } y<0 \end{cases}$$



y, x (time)

In general

$h = \frac{1}{2}(t - t_0)^2$ $t < t_0$ $\frac{dh}{dt} = -\sqrt{|h|}$

Not soln for $t > t_0$

$\frac{dy}{dt} = y^2$ $y = \frac{1}{t_0 - t}$ $y_0 = \frac{1}{t_0}$

value at $t = 0$

Domain of definition:

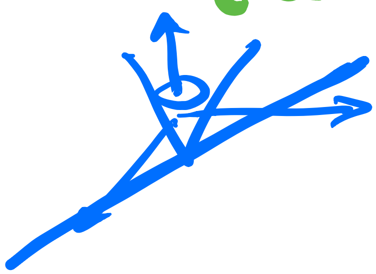
where is the soln defined

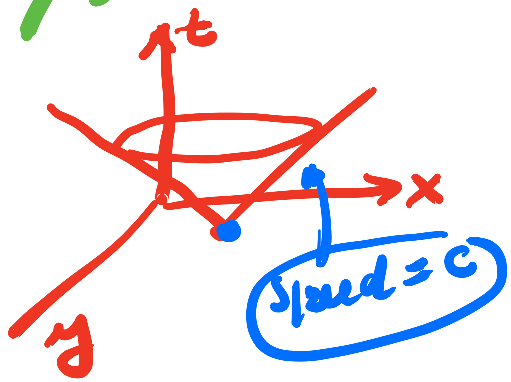
$y \to t$!

+ causality

Domain of influence

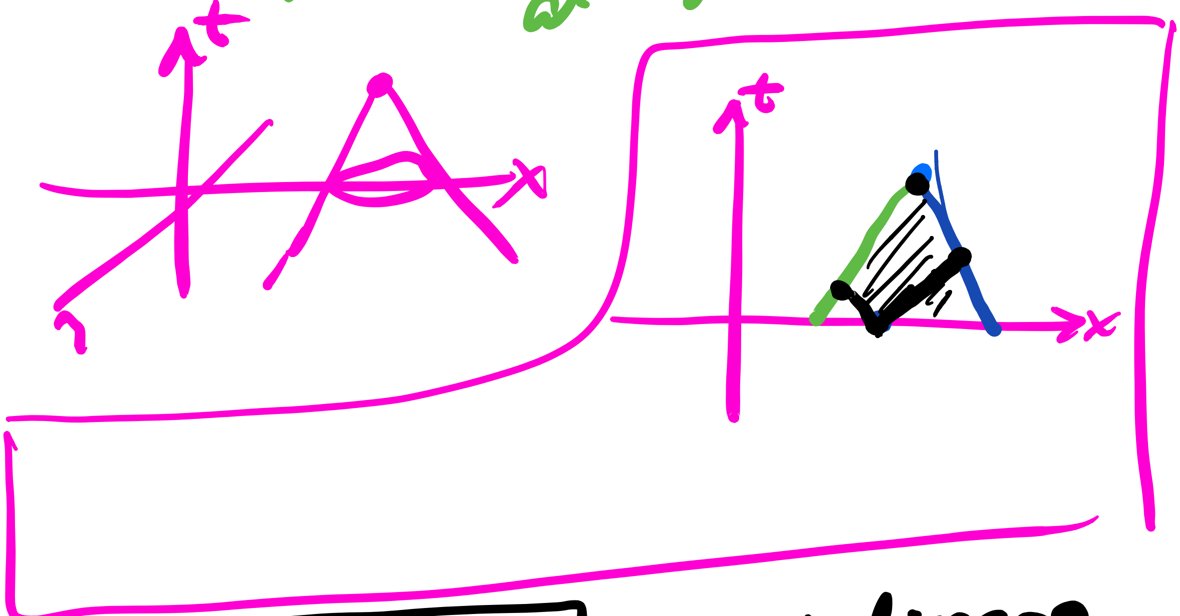
region affected by sector in the data





Domain of dependence

region that affect solution at a given point



$$\boxed{u_t + u_x = u^2}$$

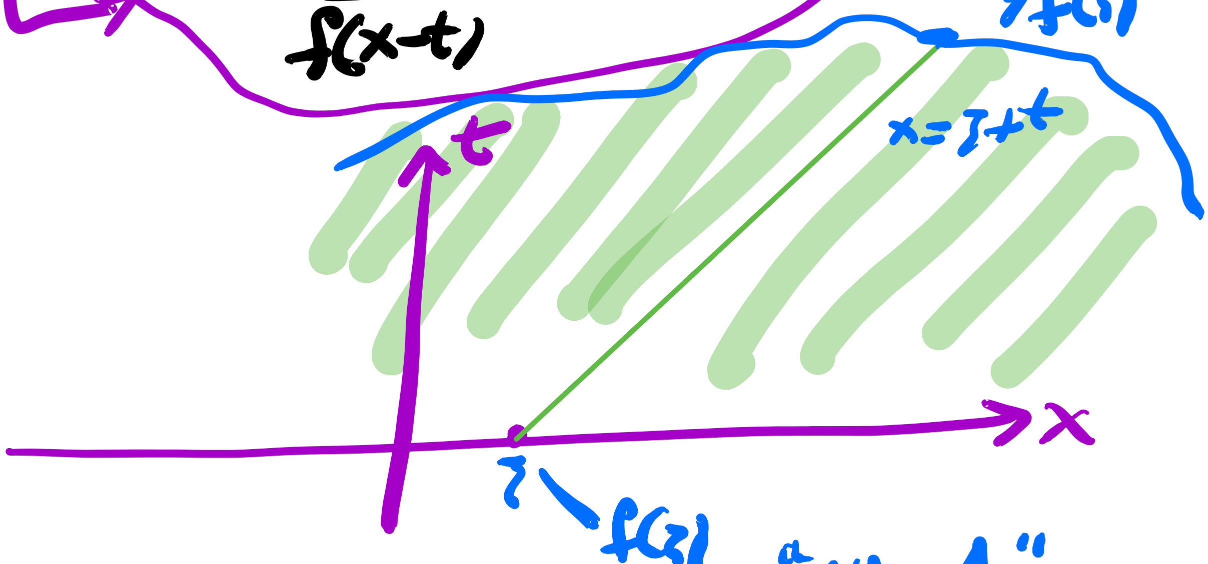
Semi-linear

$$x = \xi + t$$

$$\frac{dx}{dt} = 1 \qquad \frac{du}{dt} = u^2 \qquad u = \frac{1}{\frac{1}{f(\xi)} - t}$$

$$u\big|_{t=0} = f(x)$$

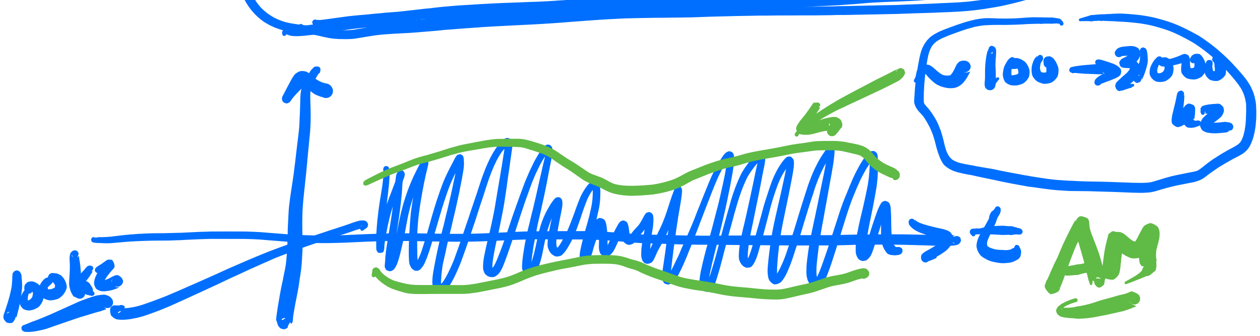
$$u = \frac{1}{\frac{1}{f(x-t)} - t}$$

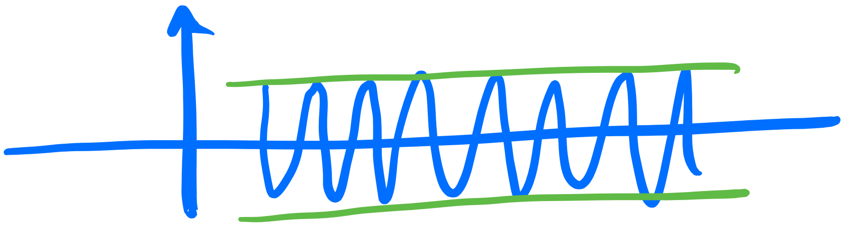


" $\psi = A$ "

Example "Laser"

$$i\psi_t = \Delta\psi + |\psi|^2\psi$$





FM

Think of waves as $\omega = \mathrm{Re}(\text{complex})$ W

$Ae^{i\omega t}$

Monochromatic

A const. ω const

$A = \rho e^{i\varphi}$

$$\boxed{W = \rho e^{i(\omega t+\varphi)}}$$

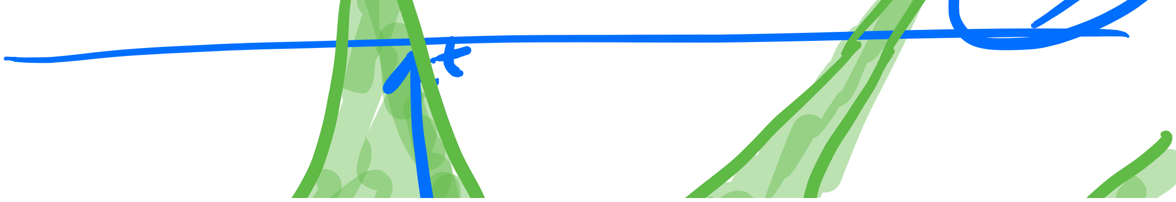
$\omega >>$

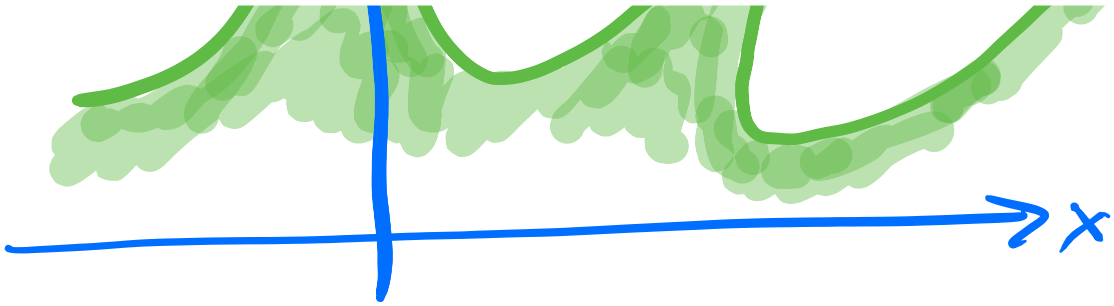
$$W = \rho(t) e^{i(\omega t+\varphi)}$$

AM

FM $\varphi = \varphi(t)$

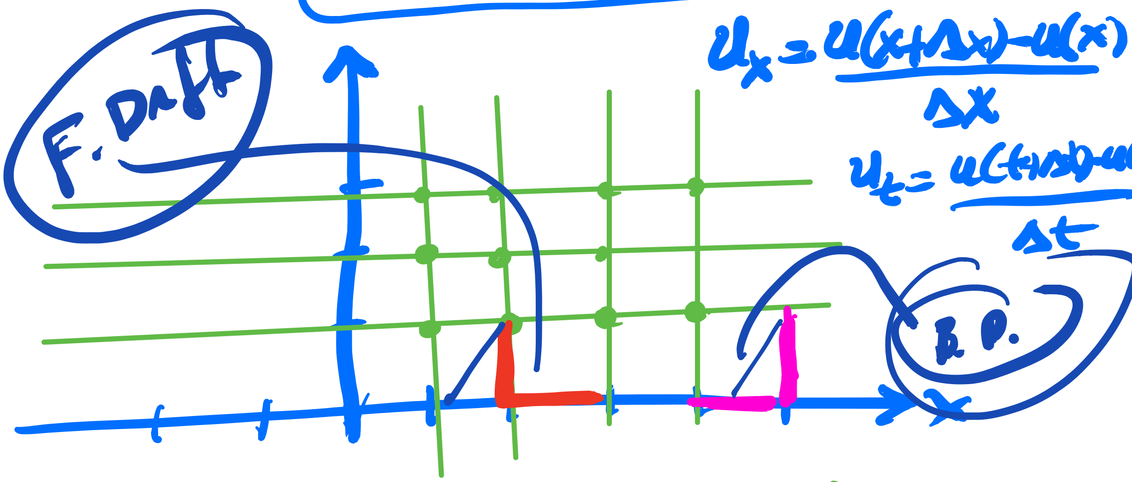
noque





Numerics

$$\boxed{u_t + u_x = f(x)}$$



$$u_x = \frac{u(x+\Delta x) - u(x)}{\Delta x}$$

$$u_t = \frac{u(t+\Delta t) - u(t)}{\Delta t}$$

$$u_m^n = u(x_m, t_n) \qquad x_m = m\Delta x$$
$$t_n = n\Delta t$$

CFL condition

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wave ω freq

k wave#

$$e^{i(kx-\omega t)}$$

$$\boxed{\begin{aligned} k &= k(x,t) \\ \omega &= \omega(x,t) \end{aligned}}$$

$\frac{k}{2\pi}$ = # of waves per unit length

$\frac{\omega}{2\pi}$ = # waves per unit time

$\boxed{\omega = \Omega(k)}$

$k_t + \omega_x = 0$

$k_t + \frac{d\omega}{dk} k_x = 0$

group vel.