$$\rho_t + \varphi(\rho)_x = (\nu \rho_x)_x$$

$$u = U(\rho) - \underbrace{k \rho_x}_{\text{preventive driving}}$$

$\boxed{\nu = \rho k}$ $\boxed{k = k(\rho)}$

If ν "small" $\Longrightarrow$ $\boxed{\rho_t + \varphi(\rho)_x = 0}$

L.W.R.

Formally time reversible

Not reversible in practice

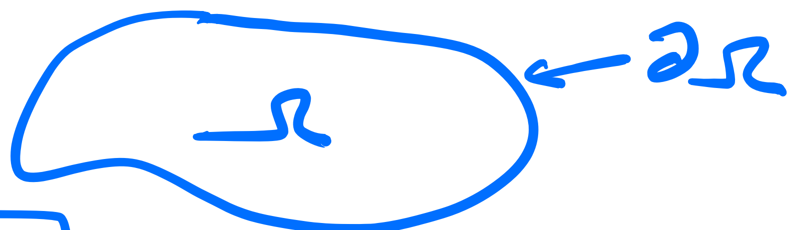
use causality

Finish Cons. Law derivation

More than 1-D

ρ density of cons. stuff.

$\vec{q}$ $\boxed{\text{flux is a vector}}$



S = source

$$\frac{d}{dt}\int_{\Omega} \rho \, dV = -\int_{\partial\Omega} \hat{n}\cdot\vec{q}\, ds + \int_{\Omega} S \, dV$$

Int. Form Cons. Law

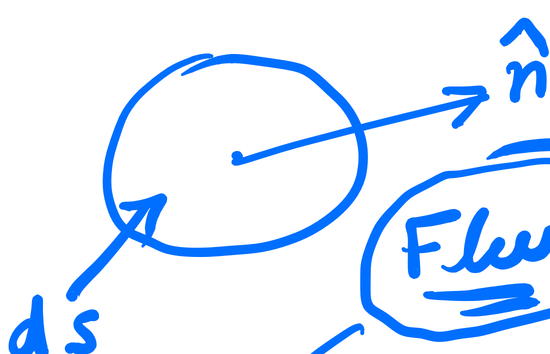
$$\int_\Omega \rho_t \, dV = -\int_\Omega \mathrm{div}\,\vec q \, dV + \int_\Omega s \, dV$$

$$\boxed{\int_\Omega \left[\rho_t + \mathrm{div}(\vec q) - s\right] dV = 0}$$

True for any Ω

$$\boxed{\rho_t + \mathrm{div}(\vec q) = s}$$

18.376/2.062 ?



Flux = $\int (\hat n)$

$\hat n \cdot \vec q$

why?

Classification of pde

→ scalar and system. #1

→ dimension 1-D/2D/3D ↦ #2

→ Order how many derivatives #3

1st order 1 derivative

$u_t + u_x = 0$

2nd order $\boxed{u_t = u_{xx}}$

$u_{tt} - u_{xx} = 0$

→ linear, semi-linear, #4
quasi-linear, ...

linear means unknown
shows up in linear
fashion only

Ex
Linear

$u_t + u_x = u, \quad u_t = u_{xx}$

$u_{tt} - u_{xx} = 0$

$$i\varphi_t = \Delta\varphi + V\varphi$$

$V = $ const. $V = V(\vec{x})$

Semilinear: It is linear in the higher order derivatives

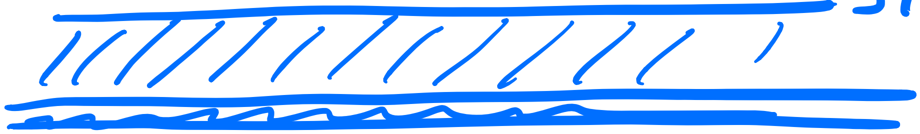
$$\boxed{u_t + u_x = f(u)}$$

Nonlinear Klein Gordon Equation

$$\boxed{u_{tt} - u_{xx} + m^2 u = 0}$$

Linear Klein Gordon

$$u_{tt} - \Delta u + m^2 u = 0$$



String

$$\boxed{u_{tt} - u_{xx} + V'(u) = 0}$$

quasi-linear

when it is linear in higher order derivatives but coeff. may depend on lower order stuff.

$$\rho_t + \Phi(\rho)_x = 0 \qquad \boxed{\rho_t + \underbrace{\frac{d\Phi}{d\rho}}_{= f(\rho)} \rho_x = 0}$$

$$u_{tt} - (c^2 u_x)_x = 0$$

$$c = c(x, u)$$

$$u_{tt} - c^2 u_{xx} - \left(\frac{dc}{du}\right) u_x u_{xx}$$

$$+ \text{———} \quad -$$

$$u_{tt} - \left(c^2(x, u_x) u_x\right)_x$$

$$\boxed{\frac{d c^2}{du_x} u_{xx}^2}$$

Finally

⊛ Hyperbolic — $u_t + u_x = 0$

"Waves" without higher order effects

⊛ Dispersive — $\boxed{u_t = u_{xxx}}$

wave speed depends on frequency

18.376 / 18.377

* **Elliptic** ~ equilibrium problems

no time

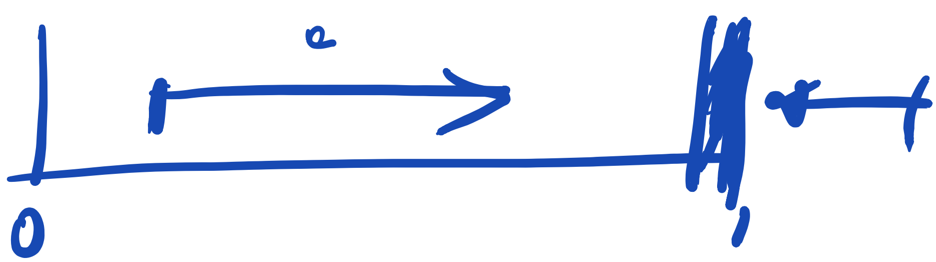
$\Delta u = 0$

Parabolic add transport effects

diffusion viscosity

$T_t = T_{xx}$

$\rho_t + \Phi(\rho)_x = (\nu \rho_x)_x$

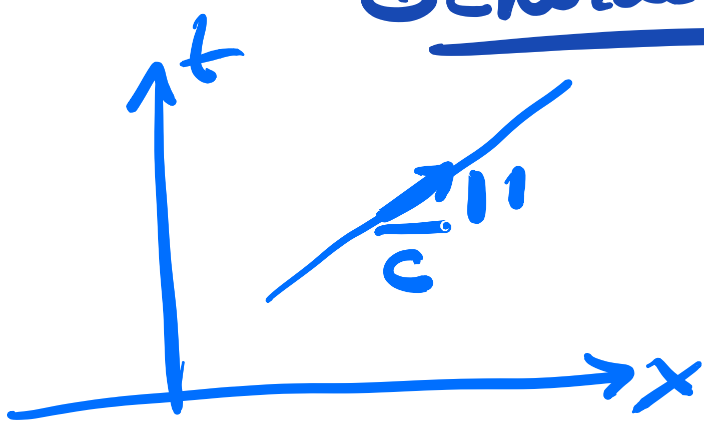


Begin Hyperbolic

$$\boxed{u_t + c u_x = a u}$$

$$u = e^{at} f(x-ct)$$

General Solution



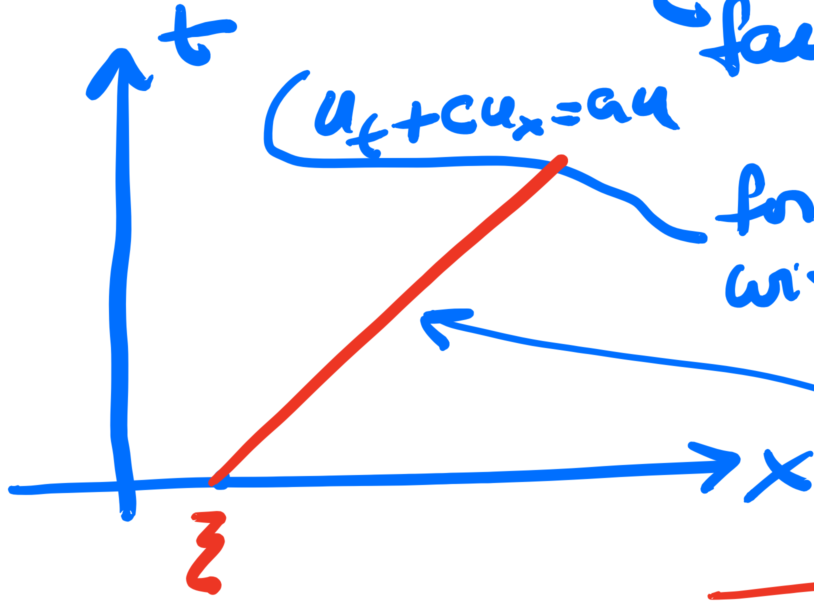
$\boxed{\frac{dx}{dt} = c}$ $\boxed{x = x(t)}$

ode.

$$\boxed{\frac{d}{dt} u(t, x(t)) = u_t + c u_x = a u}$$

$u_t + c u_x = a u$ ⟷ $\frac{dx}{dt} = c$, $\frac{du}{dt} = a u$

1 pde



$u_t + c u_x = au$

family of ode!

for $-\infty < x < \infty$ with $u(x,0) = f(x)$

$\frac{dx}{dt} = c$ with $x|_{t=0} = \xi$ $\quad \boxed{x = ct + \xi}$

$\frac{du}{dt} = au$ at $t=0$ $u = f(\xi)$

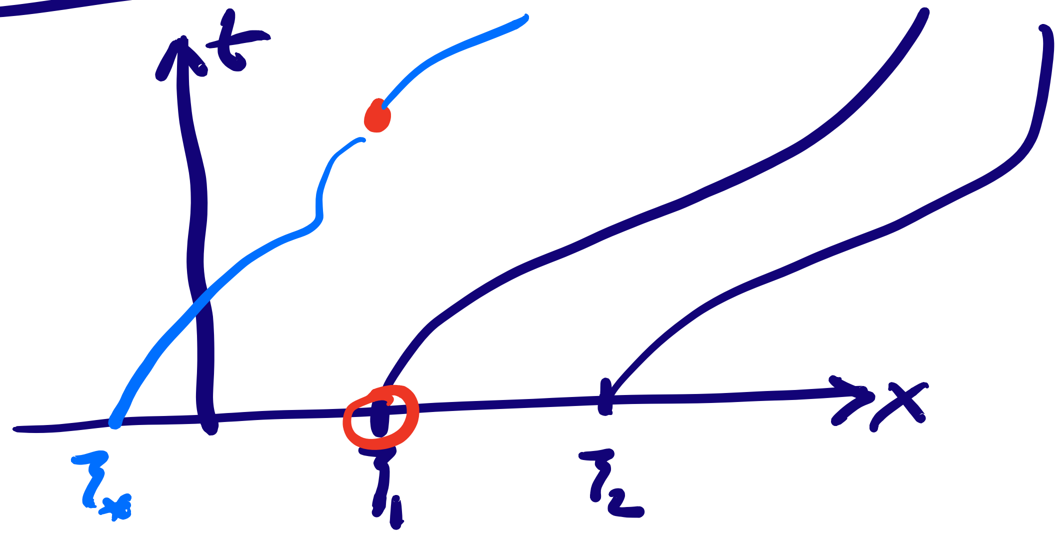
$$u = e^{at} f(\xi)$$

$\xi = x - ct$ $\quad u = e^{at} f(x-ct)$

$$u_t + c(x) u_x = g(x)$$

$-\infty < x < \infty,\ t > 0$ $\quad u(x,0) = f(x)$

$$\boxed{\frac{dx}{dt} = c(x)} \quad x\big|_{t=0} = \xi$$



$\frac{du}{dt} = g(x)$ along curve $\frac{dx}{dt} = c(x)$

at $t=0$ $u = f(\xi)$

$$\boxed{x = X(t, \xi)}$$

$$\boxed{u = U(t, \xi)}$$

Method of characteristics

$\frac{dx}{dt} = c$ sln are called $\boxed{\text{characteristics}}$

Can I get $\xi = \xi(x, t)$

X satisfies

$X_t = c(X)$ and $X(0, \xi) = \xi$

$U_t = g(X)$ and $U(0, \xi) = f(\xi)$

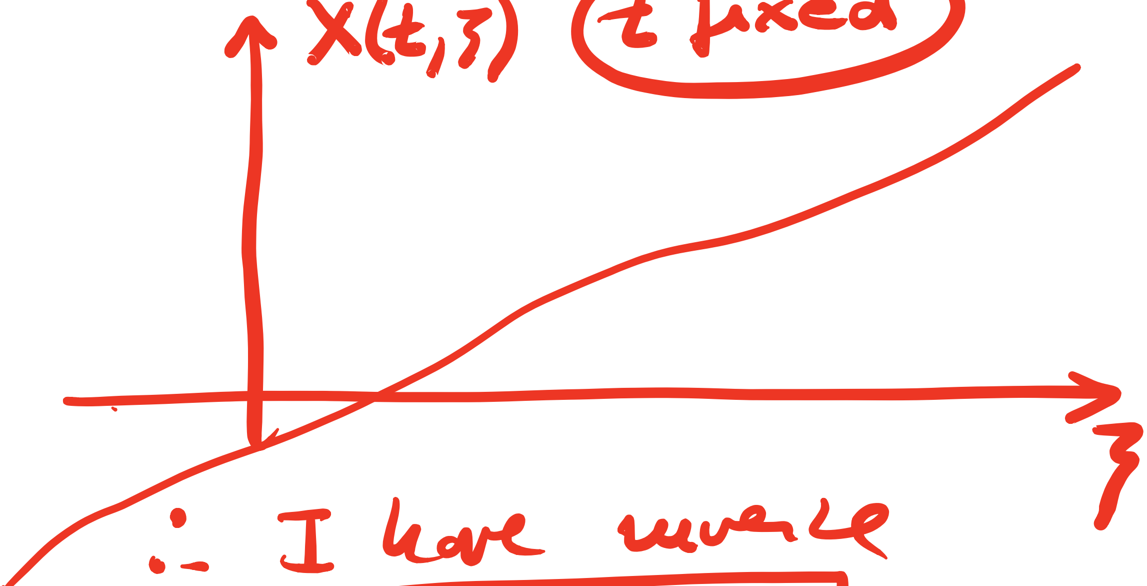
$(X_\xi)_t = (c'(X)) X_\xi$ and $X_\xi\big|_{t=0} = 1$

function of t

$\int^t c(s)\, ds$

$\dot{y} = c(t)\, y \qquad y = y(0)\, e^{\int c(s)\,ds}$

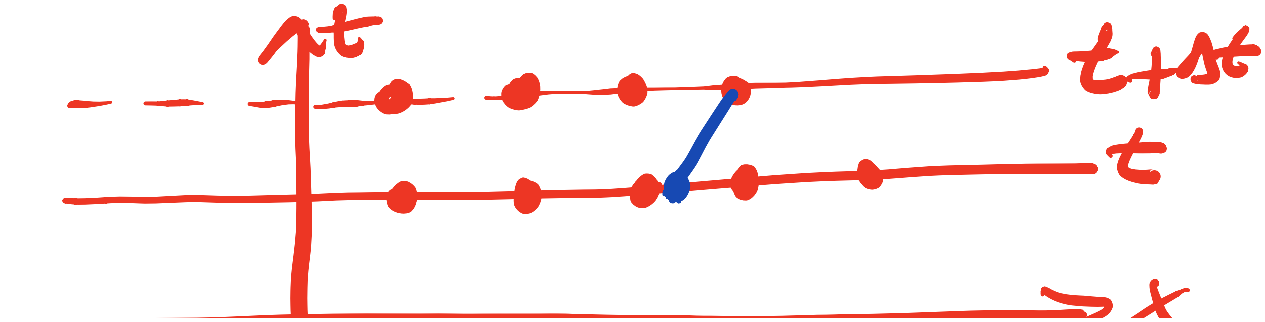
$\boxed{X_\xi > 0}$ $\boxed{\underline{\text{always}}}$



$\therefore$ I have inverse

$$\boxed{\xi = \xi(x,t)}$$

$$\Rightarrow\ u = u(\xi, t) = \underline{\underline{u(x,t)}}$$

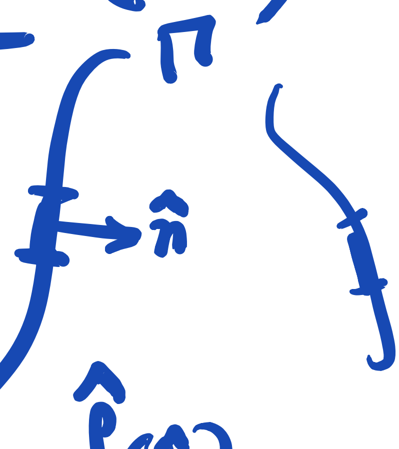


Flux is a vector. Why?

physical definition (2D)

Take a line

flux $f(\hat{n})$



$$\boxed{f(-\hat{n}) = -f(\hat{n})}$$

$\vec{f} = T \cdot \hat{n}$

$f(\hat{n}) = \vec{q} \cdot \hat{n}$