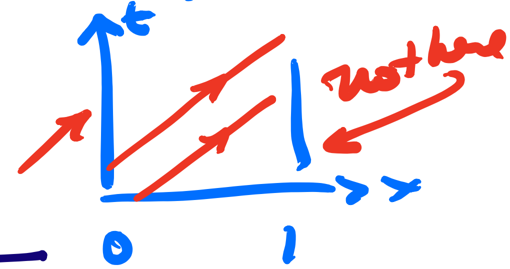
Lecture 5, Th March 4 18.306

Conservation Laws

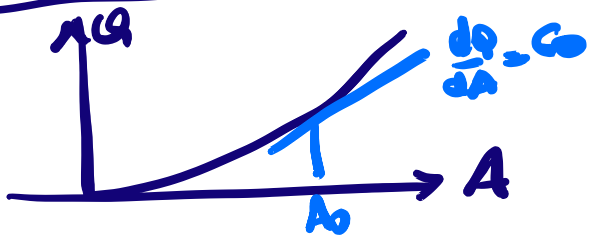
Example Causality

$u_t+u_x=0$, $0<x<1$



Example 1 Flood waves

$$\boxed{A_t+(Q(A,x))_x=0}$$



$\frac{dQ}{dA}$ positive

Flow velocity $u = Q/A$ | $Q = uA$

But $\frac{dQ}{dA}$ is also a velocity of what?

1) Assume $Q=Q(A)$
2) Assume near equilibrium $A = A_0 + \delta A$

$$Q(A) = Q(A_0) + \left(\frac{dQ}{dA}\right)_0 \delta A$$

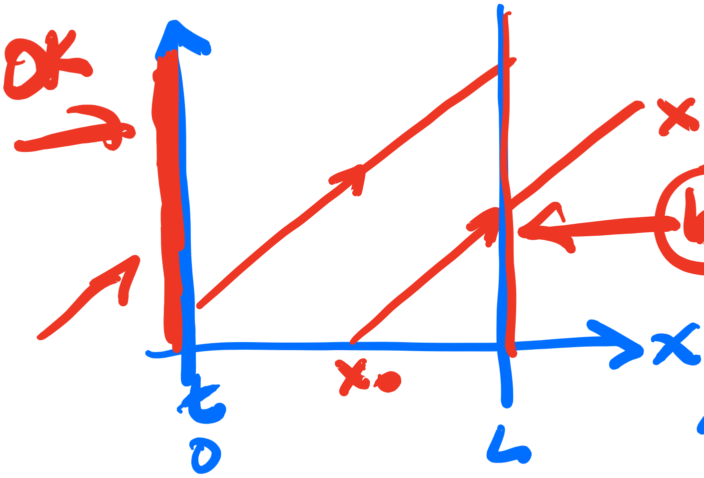
c_0

$$\boxed{(\delta A)_t + c_0 (\delta A)_x = 0}$$

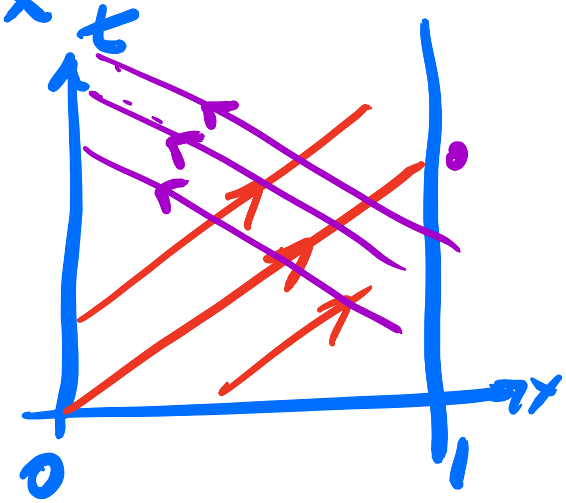
$$\delta A = f(x - c_0 t)$$

c_0 = speed of perturbation

wave speed



$x = c_0 t + x_0$

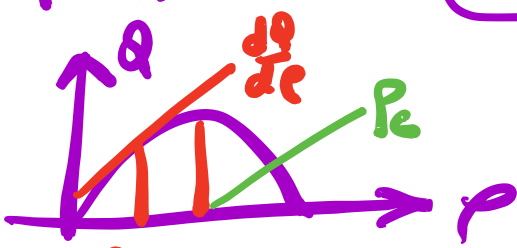


Example 2

Traffic Flow

$\rho_t + Q(\rho)_x = 0$

Q



$\rho = \rho_0 + \delta\rho$

$Q(\rho) = Q(\rho_0) + c_0\,\delta\rho$

$c_0 > 0$ if $\rho < \rho_c$

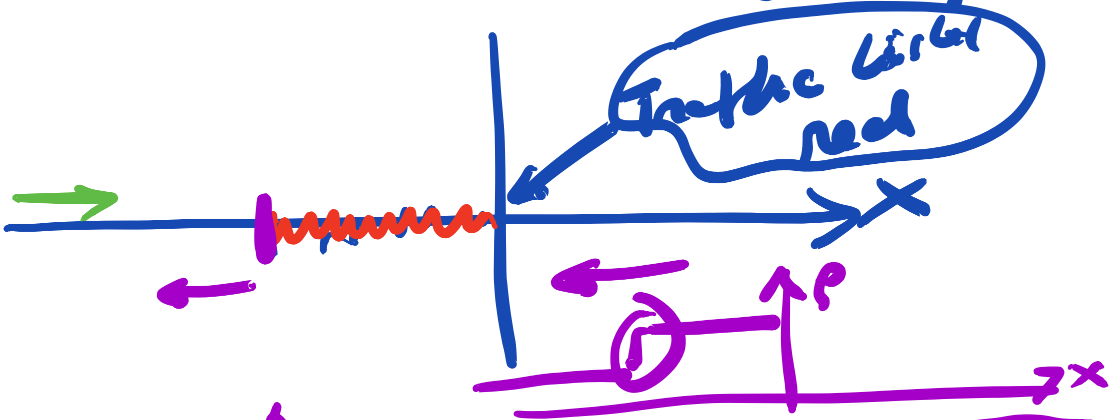
$c_0 < 0$ if $\rho > \rho_c$

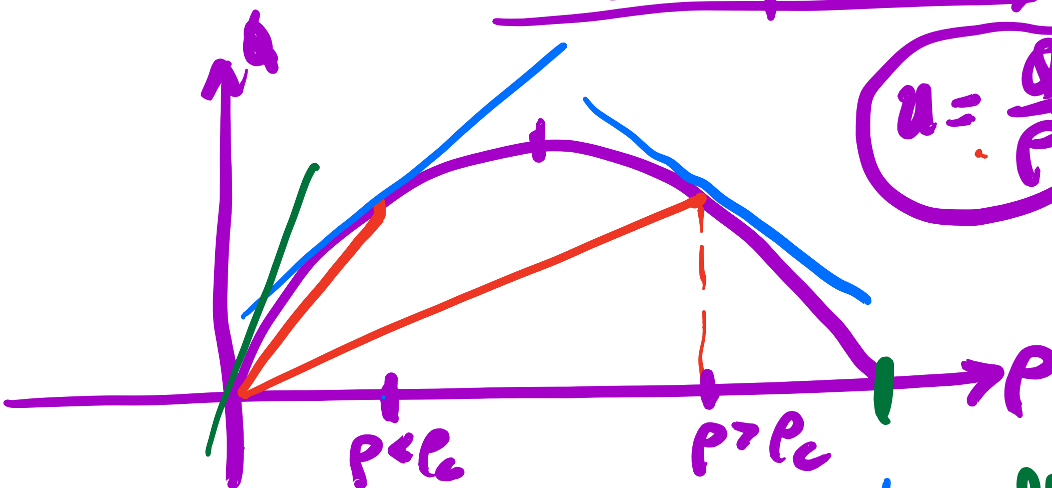
$\rho_c \longrightarrow$ $\rho < \rho_c$ | light traffic

$\rho > \rho_c$ | heavy traffic

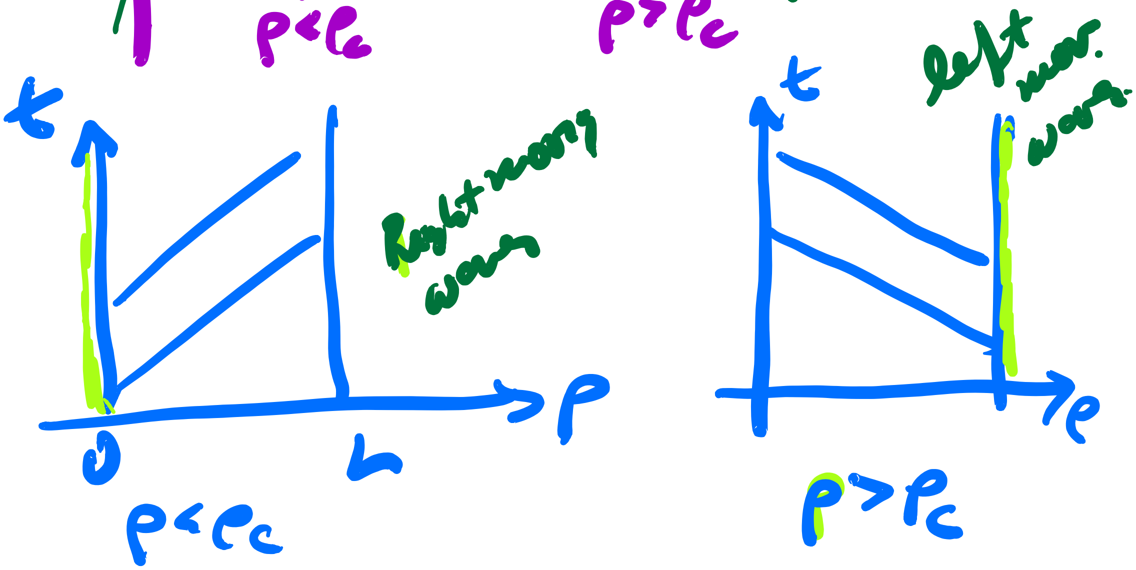
$$(\delta\rho)_t + c_0(\delta\rho)_x = 0$$

c_0 is sign "wave" speed





$$u = \frac{q}{\rho}$$



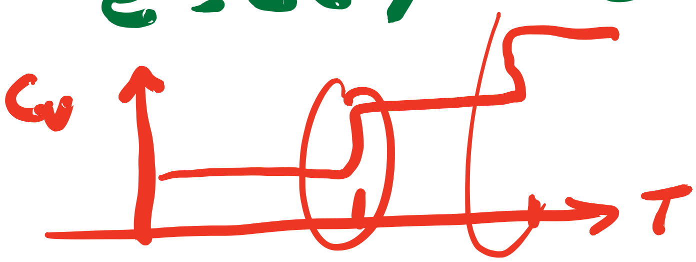
Higher order / Transport

$$\vec{u}_t + (\vec{F}(\vec{u}))_x = 0$$

Heat

$e = e(T)$ $\qquad$ $e = c_v T + e_0$

specific heat



$$e = c_v T + e_0$$

$$q_e = ?$$

$$\boxed{q_e = -\nu T_x}$$

Fick's law

$$\boxed{e_t + (q_e)_x = 0}$$

$$(c_v T)_t - (\nu T_x)_x = 0$$

Heat eqn

If c_v & ν are const

$$\boxed{T_t = \frac{\nu}{c_v} T_{xx}}$$

$\frac{\nu}{c_v}$ = heat diff. coeff.

dimension $\frac{\text{Time}}{\text{Length}^2}$

Euler Equ $\rightarrow$ Navier Stokes 1-D

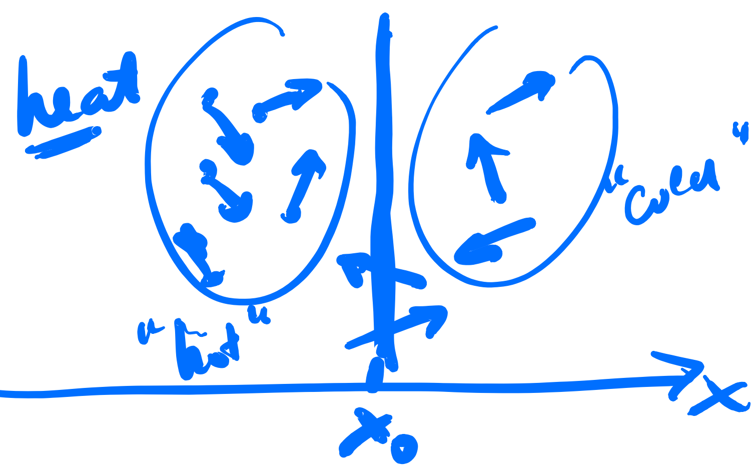
$$\rho_t + (\rho u)_x = 0$$

$$(\rho u)_t + (\rho u^2 + p - \nu u_x)_x = 0$$

$$(\rho E)_t + (\rho u E + p u - \nu u u_x - \kappa T_x)_x = 0$$

$$\boxed{E = \tfrac{1}{2}u^2 + e}$$

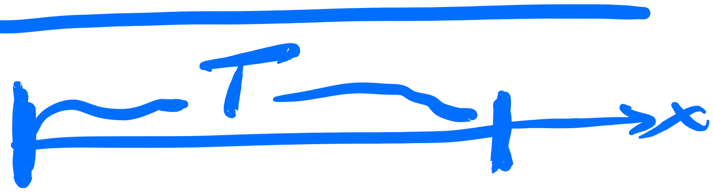
Why this



Heat Equ

$T_t = \kappa T_{xx}$

B. Cond.



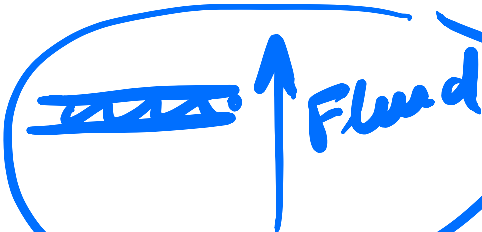
① Tell me the temperature at boundary. $T = T_b$ Dirichlet B.C.

② Tell me the heat flux

$T_x = F_b$ Neumann B.C.

③ "Cooling" at Boundary.

"Newton Law of cooling"



Flow of heat $\propto$ Diff in temp. between Fluid & solid

$$-\kappa T_x = \alpha(T_f - T)$$

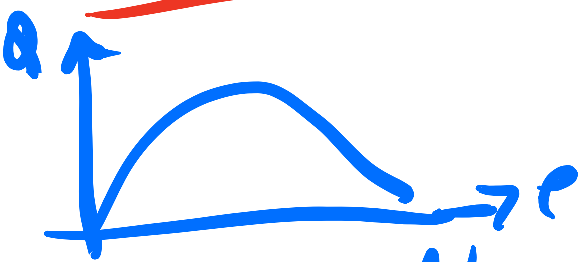
$\kappa > 0$

left side $\alpha > 0$

right side $\alpha < 0$

Traffic Flow

$$\boxed{\rho_t + q(\rho)_x = 0}$$



Quasi equilibrium

Preventative driving

$$q = Q(\rho) - \nu \rho_x, \quad \nu > 0$$

$$\rho_t + [Q(\rho) - \nu \rho_x]_x = 0 \qquad \nu = \nu(\rho)$$

$$\longleftrightarrow \quad \boxed{\rho_t + Q(\rho)_x = (\nu \rho_x)_x}$$

$$\nu = \nu(\rho, x, t)$$

Other eqns: Laplace, Poisson, Maxwell, etc