Lecture 306 2021/02/25 #3

$T_t = -T_{xx}$ $0 \le x \le 2\pi$, periodic

H.E. on a ring

$T = T_0(x)$ at $t = 0$

$T_0 = \sum_{-\infty}^{\infty} a_n e^{inx}$ $T = \sum_{-\infty}^{\infty} a_n e^{inx + n^2 t}$

need $|a_n| \le C e^{-n^2 \alpha}$ $\alpha > 0$

and then get soln

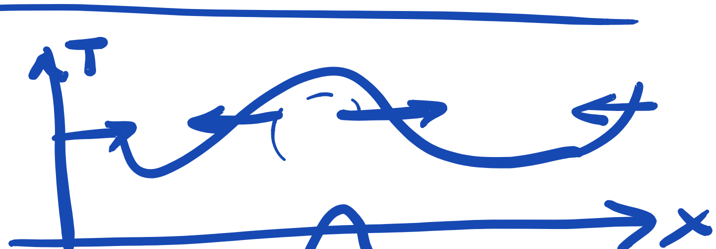
for $0 < t < \alpha$

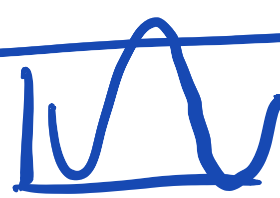
Perturbation $\delta T_0 = \epsilon e^{imx}$ $\left\{ \begin{array}{l} \text{some } m \\ 0 < \epsilon \ll 1 \end{array} \right.$

$\delta T = \epsilon e^{imx + m^2 t}$

$\epsilon e^{imx + m^2 T} = O(1)$

$T_t = T_{xx}$





$T_t = -T_{xx}$

$\sum_{-\infty}^{\infty} \longrightarrow \sum_{-N}^{N}$ ∴ largest growth rate is $e^{N^2 t}$

Other example of phenomena

#1)

Know Temperature & heat Flux

$\downarrow$ know T $\downarrow$ know T_x

Box $0 < x < L$, $0 < y < \pi$

$$\left[\begin{array}{l|l} T(0,y) = T_0(y) & T_y = 0 \text{ on} \\ T_x(0,y) = F_0(y) & y = 0, \pi \end{array}\right]$$

$$T = \sum_0^\infty a_n(x) \cos ny \qquad \Delta T = 0$$

$$0 = T_{xx} + T_{yy} = \sum \left(a_n'' - n^2 a_n\right) \cos ny$$

$$a_n'' = n^2 a_n \qquad a_n \sim e^{\pm nx}$$

$$T = \sum_0^\infty \left\{ \alpha_n \cosh nx + \beta_n \sinh nx \right\} \cos ny$$

$\sum^\infty$

$$T_0 = \sum_0 \alpha_n \cos ny$$

$$F_0 = \sum_0^\infty n \beta_n \cos ny$$

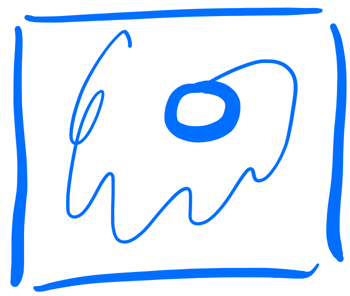
e^{nx}

#1 Image reconstruction

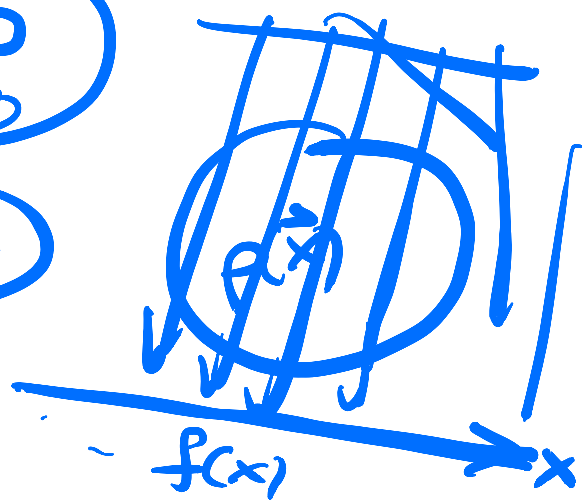
$\Delta T = 0$

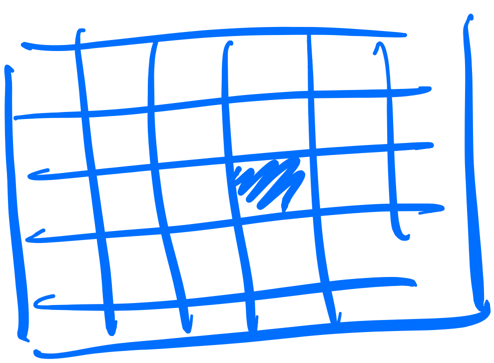
0

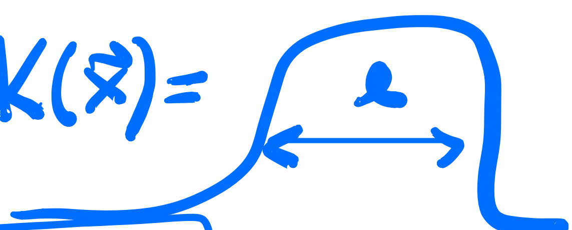
$\Delta T = 0$



#2 CAT scan

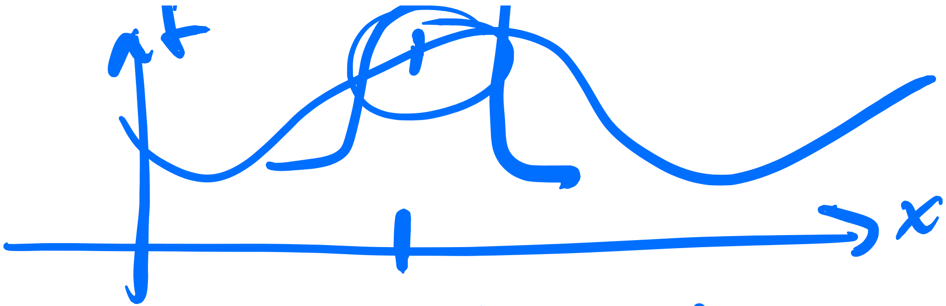




$K(\vec{x}) =$ 

$f(x)$

$$\boxed{\int K(x-y)\, f(y)\, dy} = \hat{f}(x)$$

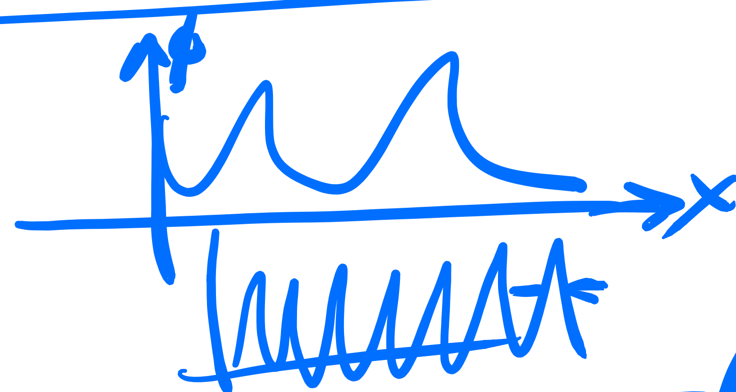


$f = \sum f_n e^{inx}$

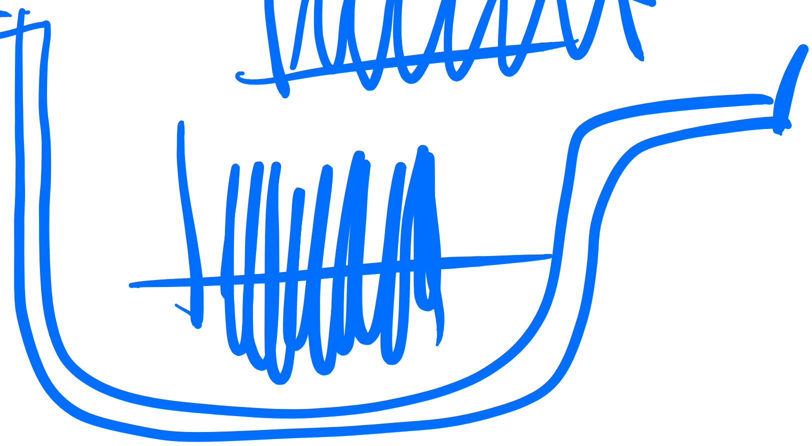
$K = \sum k_n e^{inx}$

$K * f = \sum f_n k_n e^{inx}$

$\boxed{\phi_t = N(\phi)}$



Causality



$u_t + u_x = 0$ or $u_t + u_x = -u$

flux $= U(\rho)\rho$

$$\boxed{u_t + u_x = au}$$

$0 < x < L$

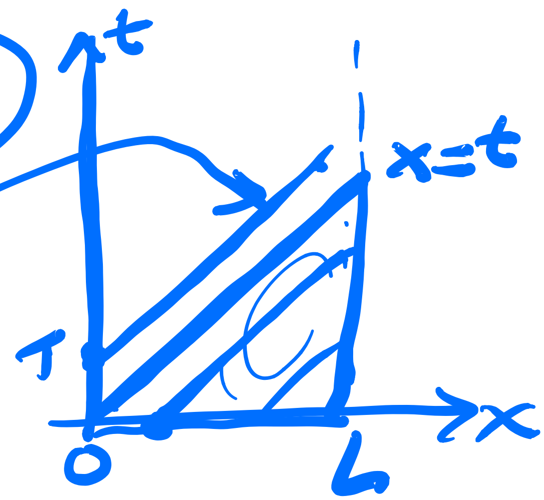
$t > 0$

$u\big|_{x=0} = \sigma(t)$

$u\big|_{t=0} = f(x)$

$u = g(x-t)e^{at}$

$x = t + x_0$



$u = f(x-t)e^{at}$ for $t < x$

$u = \sigma(t-x)e^{at}$ for $t > x$

$t > x$

$t = x + T$

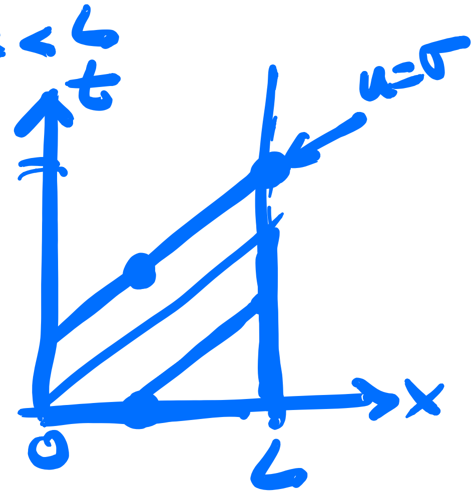
$u_t + u_x = au$

$0 < x < L$

$u(x,0) = f(x)$

$u(L,t) = \sigma(t)$

Violates causality



$$u_t + u_x = au + \underline{\epsilon u_{xx}}$$

$$\boxed{u_t = u_{xx}}$$

$u_n = u(t, x_n) \quad x_n = nh$

$$\boxed{\dot{u}_n = \frac{u_{n+1} - 2u_n + u_{n-1}}{h^2}}$$

Example

why a discretization does pde $\longrightarrow$ ode

or **FFT** $\quad u = \sum_{-N}^{N} a_n e^{inx} \Longrightarrow \underline{\dot{a}_n = -n^2 a_n}$

$$u_t = u_{xx} + (u^2)_x$$