$\dot{Y} = F(Y, t)$ $\quad$ Y = vector $\quad$ Known

$Y(0) = Y_0$

If F nice enough (e.g. has bounded derivatives) then

- soln exist
- " is unique
- " depends continuously on data over some interval $0 < t < T$
- " parameters $F = F(Y, t, \lambda)$

$$\boxed{Y = Y(t, Y_0, \lambda)}$$

N eqns and M parameters

Soln depends on $N+M+1$ variables

Example $\dot{Y} = AY$ $\quad$ $Y = e^{At} Y_0$

$\| e^{At} \| \leq e^{Kt}$ $\quad$ K constant

errors grow at a finite rate (bounded by e^{Kt})

PDE Scalar PDE $u = u(t,x)$

$$f(u_t, u_x, u, u_{tt}, u_{tx}, \dots) = 0$$

$u_t = 0$ and at $t=0$ $u = g(x)$ | $\boxed{u = g(x)}$

$u_t + u_x = 0$
$u = g(x)$
at $t=0$

$(x,t) \to (x-t, t)$ ξ

$u = u(x-t, t)$

$u_t \to u_t - u_\xi$ | $u_t = 0$

$u_x \to u_\xi$

$$\boxed{u = a(x-t) = g(x-t)}$$

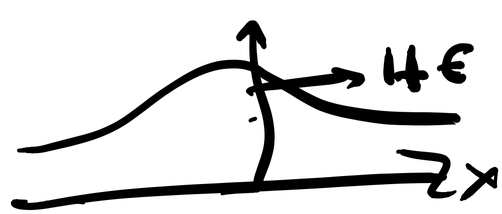
$u_t + c u_x = 0$

$u = g(x)$ at $t=0$

$\xi = x - ct$

equ $\to$ $u_t = 0$

sln $u = g(x-ct)$



$$u_t + c u_x = a u$$

$\xi = x - ct$

$u_t \to u_t - c u_\xi$

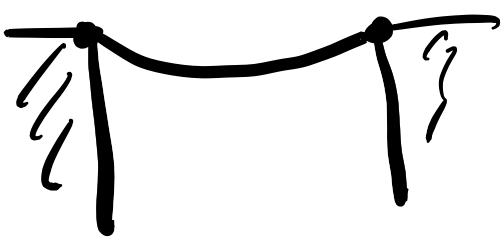
$u_x \to u_\xi$

$\to \boxed{u_t = a u}$ $\quad u = u(\xi, t)$

$$u = e^{at} u(\xi, 0) = g(x-ct) e^{at}$$

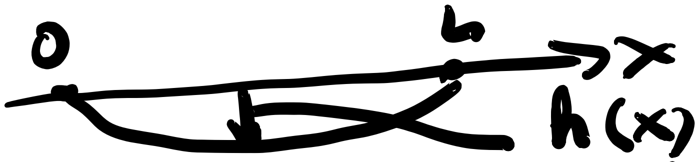
$$\boxed{u_t + f(u_x) = a u^2}$$

$\dot{Y} = F(Y)$ $\qquad u_t = F(u, u_x, \dots)$



hanging bridge

What is the shape



What is the shape of a hanging chain?

Models
$$\boxed{\begin{array}{l} F(h, h_x, h_{xx}) = 0 \\ \text{and } h(0) = 0 \quad h(L) = 0 \end{array}}$$

Boundary Value Problem BVP

$\dot{Y} = F(Y)$ Initial Value Problem IVP

$F(h, h_x, h_{xx}) = 0 \quad 0 < x < L$

$h(0) = 0, \quad h(L) = 0$

1) $u'' + \pi^2 u = 0 \quad 0 < x < 1$

$u(0) = 0 \quad u(1) = 0$

$u(0) = 0$

$\Rightarrow u = a \sin \pi x$

∞ sln

IP

2) $u'' + u = 0 \quad 0 < x < 1$

$u(0) = u(1) = 0$ 1 sln

UP

$u = a \sin x$

$u(1) = 0 \Rightarrow a = 0$

$u = 0$

3) $u'' + \pi^2 u = 0 \quad 0 < x < 1$

$u(0) = 0, \quad u(1) = 1$

IP

$u = a \sin \pi x \quad u(1) = a \sin \pi = 0$

No sln

Desirable properties of a problem

1. has only one soln.
2. soln depends on data continuously

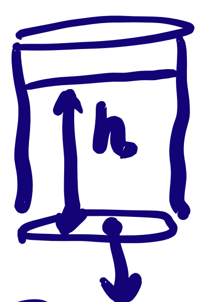
Well Posed Problem

$\boxed{\text{Ill posed means either (1) or (2) or both fail}}$

Examples of pde & ode ill posed

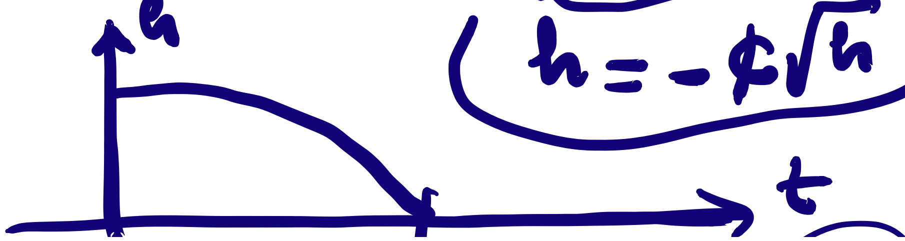
physical

leaky bucket



$h = h(t)$

$$\dot{h} = -c\sqrt{h}$$



t_c'

$\dim c = \left(\frac{\sqrt{c}}{\text{time}}\right)$

$$\frac{dh}{dt} = -\sqrt{h} \qquad h \geq 0$$

$$\frac{1}{\sqrt{h}}\, dh = -dt \qquad 2\, d\sqrt{h} = -dt$$

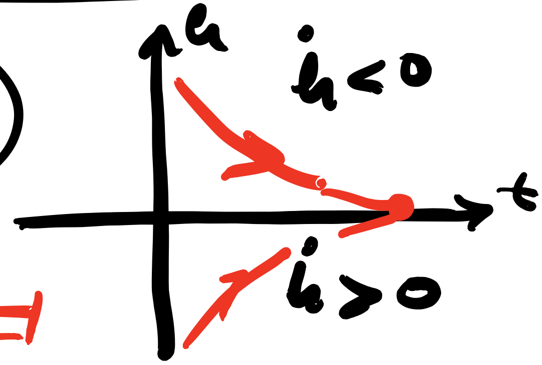
$$2\sqrt{h} = t_0 - t$$

$$h = \frac{1}{4}(t_0 - t)^2 \qquad h_0 = \frac{1}{4} t_0^2$$

$$\boxed{\dot{h} = \sqrt{h}}$$

$h(0) = 0$

$$\frac{dh}{dt} = -(\operatorname{sign} h)\sqrt{|h|}$$



$$\frac{dh}{dt} = -(\operatorname{sign} h)\,|h|^{0.9}$$

dt

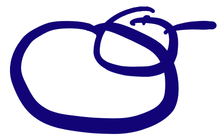
$|h|^{1.1}$ ←

$\dot{h} = f(\text{sign } h)\,|h|^{\alpha}$

$\alpha > 1$ ok

$\alpha < 1$ bad

P.D.E. problems



$T = T(x,t)$

$0 < x < 2\pi$

periodic

$\boxed{T_t = \nu T_{xx}}$ heat eqn on a ring

slns periodic in x of period 2π

ν = diffusivity constant

$T(x,0) = T_0(x)$ simplify $\nu = 1$

$T_0(x) = \sum_n a_n e^{inx}$ $a_{-n} = a_n^*$

$e^{inx - n^2 t}$

$T(x,t) = \sum_n a_n e^{\ldots}$

Thermal archeology is it possible?

$T_t = -T_{xx}$ periodic

$T(x,0) = T_0(x)$

$$\boxed{T(x,t) = \sum a_n e^{inx + n^2 t}}$$

Cauchy-Kowalewski

$u_t = F(u, u_x, \ldots)$