

18.306 Advanced Partial Differential Equations with Applications.

Tue & Thu 9:30–11:00 (virtual)

Covid-19: This course will be fully virtual, with (live) online lectures, which will be recorded and accessible through canvas. The course will have two web pages: one public in the Math. site, and another in canvas [used to upload problem set answers, access class recordings, announcements, etc.] Because the situation is somewhat unpredictable, this syllabus may have to be updated later in the semester.

Description: The focus of the course are the concepts and techniques for solving the pde (partial differential equations) that permeate various scientific disciplines. Applications include problems from fluid dynamics, electrical and mechanical engineering, materials science, quantum mechanics, etc.

Prerequisites: Basic theory of complex variables and a good working knowledge of ode.

Textbook: None required. However, the hyperbolic equations part is inspired/based by chapters 1–10 in Whitham’s book, while the rest relies on chapters 1–5 of Salsa’s book — see the textbook list below. Supplementary notes will be posted on the web page; however, these notes are **not** inclusive.

Instructor: R. R. Rosales, rrr@math.mit.edu. Off. hrs TBA later.

TA: TBA.

Exams: None. Neither midterm, nor final. See **GRADING** below.

PROBLEM SETS: 6 ± 1 problems sets (one every 1-2 weeks). **Do them all:** *you need them to learn the material.* Must be handed in pdf format through the canvas website. See **GRADING** below.

Math. site WEB page: <http://math.mit.edu/classes/18.306/index.html>

MatLab: I strongly urge you to become proficient in MatLab. See the course web page for more information. You can install MatLab in your own computer. See: <https://ist.mit.edu/matlab/all/student>

News Updates: *Check the News Updates link in the web page frequently!*

Information concerning office hours, TA, etc. will be posted there.

GRADING: Each problem set will contain a buried mini-quiz within it (1-3 problems). Only the quiz will be graded, but you need to do all the problems, since the quiz problems will not be identified. The course grade will be based on the cumulative quiz-grade, and a binary-grade term paper.

Term paper grading: If you hand in an acceptable term paper, then the grade is quiz-based, as explained above. Else the grade is F. **The term paper should be handed one week before the last lecture.**

Note: **complete answers to the problem sets will be posted after their due date.** Compare your solutions (even those not graded) with the one posted. If after this you have doubts regarding your answers, consult with the TA and/or the lecturer. You should not be left wondering if you answer was correct or not.

TERM PAPER. On any topic relevant to the course materiel (instructor pre-approval required). It need not be original research, but it must be original work [e.g.: literature review in some topic, with summary in your own words, giving proper credit to the sources]. *The explanations must be clear and accessible to this class average student.* You can use materiel from your own research, but “recycling” (handing a piece of your thesis) is not allowed. You must process it to follow the guidelines here. Further requirements:

- Must be typed (font size 12-14) and submitted electronically **in pdf format**.
- Length below 13 pages, with standard page formatting, not including figures.

Failure to follow these guidelines may result in a paper being classified as “not acceptable”.

Recommended books and textbooks (all on library reserve).

1. Whitham, G. B. *Linear and nonlinear waves*. Wiley.
2. Salsa, S. *Partial differential equations in action*. Springer.
3. Debnath, L. *Nonlinear partial differential equations for scientists and engineers*. Springer.
4. Guenther, R. B., and Lee, J. W. *Partial Differential Equations of Mathematical Physics and Integral Equations*. Dover.
5. Levine, H. *Partial differential equations*. Am. Math. Soc.: International Press, 1997.
6. Kevorkian, J. *Partial differential equations: analytical solution techniques*. Springer.
7. Carrier, G. F., and C. E. Pearson. *Partial differential equations: theory and technique*. Ac. Press.
8. Hinch, E. J. *Perturbation methods*. Cambridge U. Press.
9. Barenblatt, G. I. *Scaling, self-similarity, and intermediate asymptotics*. Cambridge U. Press.
10. Drazin, P. G., and R. S. Johnson. *Solitons: An Introduction*. Cambridge U. Press.
11. John, F., *Partial Differential Equations*. Springer. #
12. Evans, L. C. *Partial Differential Equations*. Am. Math. Soc. #
13. DiBenedetto, E. *Partial Differential Equations*. Birkhauser. #
14. Garabedian, P. R. *Partial Differential Equations*. Am. Math. Soc. #

The books marked with # emphasize rigorous mathematical theory. For many of the books there is more than one edition, but this has no significance for the actual course material. When a problem is assigned from a book, the full problem statement will be displayed in the assignment, to avoid possible conflicts between editions.¹

Course topics. Some things may be covered in more detail than this implies, or the reverse. It is unlikely that there will be enough time to cover everything. Topics not here may be added later.

- **Introduction.** Terminology; boundary and initial value problems; well- and ill-posed problems.
- **First-order pde.** Complete solutions; characteristics; conservation laws; systems of pde; introduction to weak solutions: shocks and jump conditions; entropy condition; examples: traffic flow, gas dynamics, etc.
- **Linear pde.** Review and classification; Laplace, wave and diffusion equations; Klein-Gordon equation; more on characteristics; standard methods: separation of variables, integral transforms, Green’s functions; potential scattering; special topics in conformal mapping; dispersion and diffusion; dimensional analysis and self-similarity; regular and singular perturbation theory; asymptotics for complete solutions; geometrical optics and WKB. eikonal equation; high-frequency expansions; caustics;
- **More on nonlinear pde.** Equations that convert into linear pde; some exactly solvable cases; Burgers’ equation; dimensional analysis and similarity; traveling waves; nonlinear diffusion and dispersion; the KdV, nonlinear Schrödinger and Sine-Gordon equations; reaction-diffusion equations; Fisher’s equation; singular perturbations: boundary layers, homogenization, weakly nonlinear geometrical optics, etc.; Solitons; Backlund transformations; Painlevé conjecture.
- **Variational Methods.** First and second variation; Euler-Lagrange equation; constraints.
- **Free-boundary value problems.** Formulation; perturbation theory; more on water waves; method of extended gradient; materials surface evolution; some open problems.

¹ The edition I own is, almost always, not the latest!