To solve the problem you take Fourier transform. I assume $m \in \mathbb{R}$ ($m^2$ is the potential so usually is real) so that $-|\xi|^2 - m^2 = 0$ has solution only when $m = 0$, and the solution is $\xi = 0$. You can either use that bounded (since we assume decay at infinity) harmonic function must be constant or functions with Fourier transform supported at 0 are polynomials. But these are much much deeper fact. Then you can discard any non-zero solutions due to the boundary condition at infinity. Some of you write that $\hat{u}$ is the Dirac delta measure at 0. This is not correct, $\hat{u}$ can be distributional derivatives of $\delta_0$ of any order, and the order corresponds to the degree of the monomial in the physical space. No penalty is given to the homogeneous part as long as you get only trivial solution and every does.

The explicit expression (not in terms of Fourier transform) of Green’s function is quite hard to find by hand, so leaving the Fourier integral is perfectly fine. The uploaded solution has detailed computation which is pretty good. But I highly recommend you do it at least on computer because knowing the kernel helps you know more about the equation. For this problem you can Google Yukawa potential.