4. Let $f$ be an upper semicontinuous function on compact metric space $X$. Then \( \{f < a_3\} \) is open $\forall a \in \mathbb{R}$, and $\bigcup \{f < a_3\}$ is an open cover of $X$ (as $f$ is given to be real-valued). If $\{a_n\} \subset \mathbb{R}$ such that $f < a_n$ is a finite subcover, then $\sup x_i = M$, then $X = \bigcup (f < a_3) \leq \{f < M^3\}$, which implies $\exists i \in \mathbb{N}$ that $f$ is bounded. Let $s$ denote the supremum of $f$ over the domain $X$, and consider the sets $\{f < s - \frac{1}{n}\}$ for $n \in \mathbb{N}$. Clearly, $\{f < s - \frac{1}{n}\} \supset \{f < s\}$, but we have also that $\{f < s - \frac{1}{n}\}$ is closed and therefore compact $\forall n$. By the corollary theorem 2.36 in W. Rudin, Principles of Mathematical Analysis, page 38, $\bigcap \{f < s - \frac{1}{n}\}$ is nonempty — which implies that $\exists x \in X$ that achieves its maximum, as desired.

5. $a \in \mathbb{R}$, \( \{g < a_3, g > a_3\} \) are countable unions and/or countable intersections of the sets $A_i$ for $j$ and $B_i$ for $h$, respectively, and the Borel $\sigma$-algebra is closed under countable unions and intersections. It follows that $g^{-1}\{(g, \beta)\} = \{g > a_3 \cap g < \beta\}$, $h^{-1}\{(g, \beta)\} = \{h > a_3 \cap h < \beta\}$ are Borel for all segments $(a_3, \beta)$ since all open sets $V$ may be written as a countable union of such segments, $g^{-1}(V)$ and $h^{-1}(V)$ are Borel for all open $V$. This implies that $g$ and $h$ are Borel because the Borel field is exactly the $\sigma$-algebra generated by all the open sets, as desired.