We first prove the hint. Let $g = \sum a_i \cdot 1_{E_i}$ be a simple function, with $1E_i$ disjoint. Then

$$\int g \, d\nu = \sum_{i=1}^{n} a_i \int 1_{E_i} \, d\nu = \sum_{i=1}^{n} a_i \int 1_{E_i} \, d\nu = \sum_{i=1}^{n} a_i \nu(E_i)$$

Now let $g$ be a nonnegative measurable function. Then $\int f \, d\mu$ is a sequence of nonnegative functions $\phi_n$, $\phi_n \leq g$ (by pg. 44 of Stroock). As $f$ is nonnegative and measurable, $\phi_n \uparrow f$, and thus by the MCT,

$$\int g \, d\nu = \lim_{n \to \infty} \int \phi_n \, d\nu \leq \lim_{n \to \infty} \int \phi_n \, d\nu$$

$$= \lim_{n \to \infty} \int \phi_n f \, d\mu$$

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$$= \int f \, d\mu.$$