HW1.1 Let $J = [a,b] \subset \mathbb{R}$ and let $a_0, \ldots, a_n$ satisfy $a = a_0 < a_1 \cdots < a_n = b$.
Assume $\psi : J \rightarrow \mathbb{R}$ is constant on each interval $(a_m, a_{m+1})$, $0 \leq m \leq n - 1$.
Show that if $\phi \in C(J)$, then $\phi$ is $\psi$-Riemann integrable and

$$(R) \int_J \phi(x) d\psi(x) dx = \sum_{m=0}^{n} \phi(a_m) d_m,$$

where the numbers $d_m$ are given by

$d_0 = \psi(a)-\psi(a)$, $d_m = \psi(a_m+)-\psi(a_m-)$, for $1 \leq m \leq n-1$, $d_n = \psi(b)-\psi(b-)$.

HW1.2 Stroock 1.1.9
HW1.3 Stroock 1.2.26
HW1.4 Suppose $\psi : J \rightarrow \mathbb{R}$ is non-decreasing and $f : J \rightarrow \mathbb{R}$ is $\psi$-Riemann integrable. Assume $m \leq f \leq M$ on $J$ and $\phi : [m,M] \rightarrow \mathbb{R}$ is a continuous function. Prove $\phi \circ f : J \rightarrow \mathbb{R}$ is $\psi$-Riemann integrable on $J$.

HW1.5 Let $\psi : J \rightarrow \mathbb{R}$ be a non-decreasing function. Let $p, q \in \mathbb{R}$ be positive numbers satisfying

$$\frac{1}{p} + \frac{1}{q} = 1.$$

Prove:

(a) If $u, v \geq 0$, then

$$uv \leq \frac{u^p}{p} + \frac{v^q}{q}.$$

(b) If $f, g \geq 0$ are $\psi$-Riemann integrable functions on $J$, then

$$\left| \int_J fg d\psi \right| \leq \left( \int_J f^p d\psi \right)^{1/p} \left( \int_J g^q d\psi \right)^{1/q}.$$

(Hint: If both integrals on the right hand side are non-zero, renormalize the problem so they are both equal to 1 and use part (a).)