Knots and Numbers: Homework
18.095, January 15, 2016
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1. The strands of a rational tangle, taken individually, are obviously unknotted. Prove the converse or find a counterexample.

2. Any rational number is of one of three forms: odd/even, even/odd, or odd/odd. What is the corresponding breakdown of rational tangles? How should ∞ be handled?

3. If K is a rational tangle with corresponding number q, what is the rational number corresponding to the mirror image of K?

4. (A little harder) Prove that $R^2 = I$ on rational tangles. (This must be the case if the Schubert-Conway theorem is true, since $R^2 q = q$ for any rational number.) It may help to consider at the same time the operation of rotating the tangle by $180°$ around a horizontal axis.

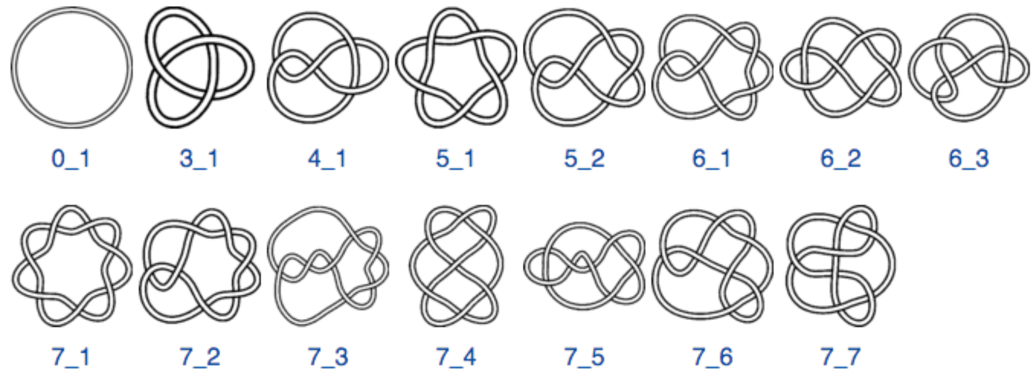
5. (For number theorists) Explain why getting from 0 to a rational number q by means of the operations R, T, and T^{-1}, can be accomplished by expressing q as a *continued fraction*. You may want to express the continued fraction using negatives:

$$a_0 - \cfrac{1}{a_1 - \frac{1}{a_2 - \cdots}}$$

6. A "two-bridge link" is a link that can be presented with just two highest points and two lowest points. Explain why such links are the same as links obtained by tying NE to NW and SE to SW in a rational tangle. Draw pictures of $T^n 0$ and some other examples. It turns out that all prime knots with fewer than 8 crossings are 2-bridge knots. Can you draw pictures of them showing that? It turns out that exactly 12 of the 21 8-crossing prime knots are 2-bridge knots. I think it's rather hard to pick them out from the pictures!

Different rational tangles may produce the same link by this process. Come up with a list of modifications of a rational tangle that don't change the corresponding link. A complete list will lead to a complete invariant for two-bridge links.

Knots with 7 or fewer crossings



Knots with 8 crossings