18.095 **IAP 2016**

Problem Set 1 Part 2 **Due: 01/8/2016**

Problem 1. If $A^{\mathrm{T}}Av = \sigma^2 v$, multiply both sides by A and *put in parentheses* to show this again: **If v is an eigenvector of $A^{\mathbf{T}}A$, then Av is an eigenvector of $AA^{\mathbf{T}}$.**

Problem 2. Find the eigenvalues and unit eigenvectors of $A^{\mathrm{T}}A$ and AA^{T}. Keep each $Av = \sigma u$. Then construct the singular value decomposition and verify that A equals $U\Sigma V^{\mathrm{T}}$.

$$\textbf{Fibonacci matrix} \quad A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Problem 3. Use the **svd** part of the **MATLAB** demo **eigshow** to find those v's graphically (from Problem 2).

Problem 4. Suppose A is invertible (with $\sigma_1 > \sigma_2 > 0$). Change A by ***as small a matrix as possible*** to produce a singular matrix A_0. Hint: U and V do not change:

$$\text{From} \quad A = \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & \\ & \sigma_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix}^{\mathrm{T}} \quad \text{find the nearest } A_0.$$