a journey into the world of

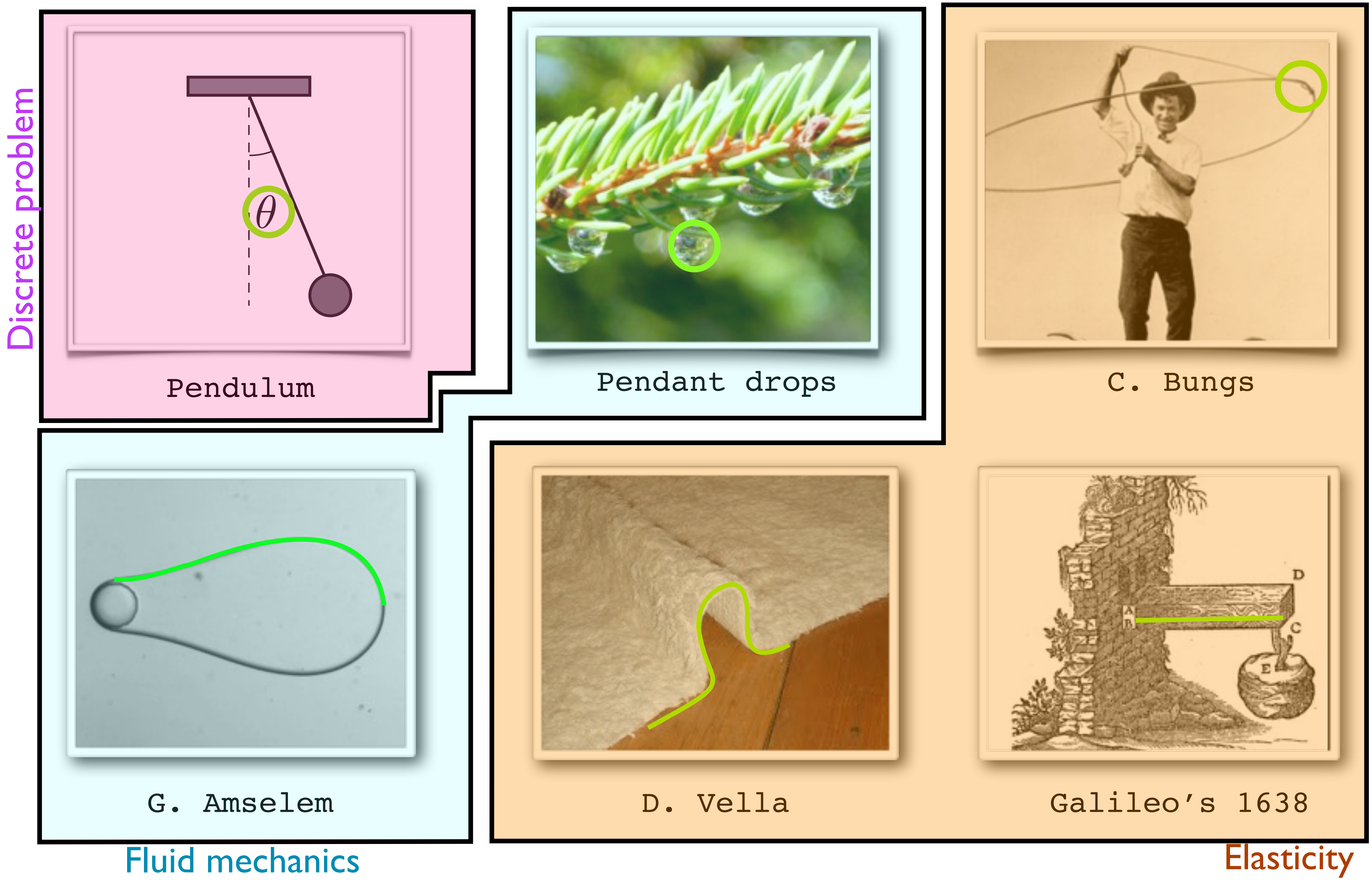
EULER'S ELASTICA

when rods, droplets and pendula behave the same.

Mathematics Lecture Series 2015

P.-T. BRUN

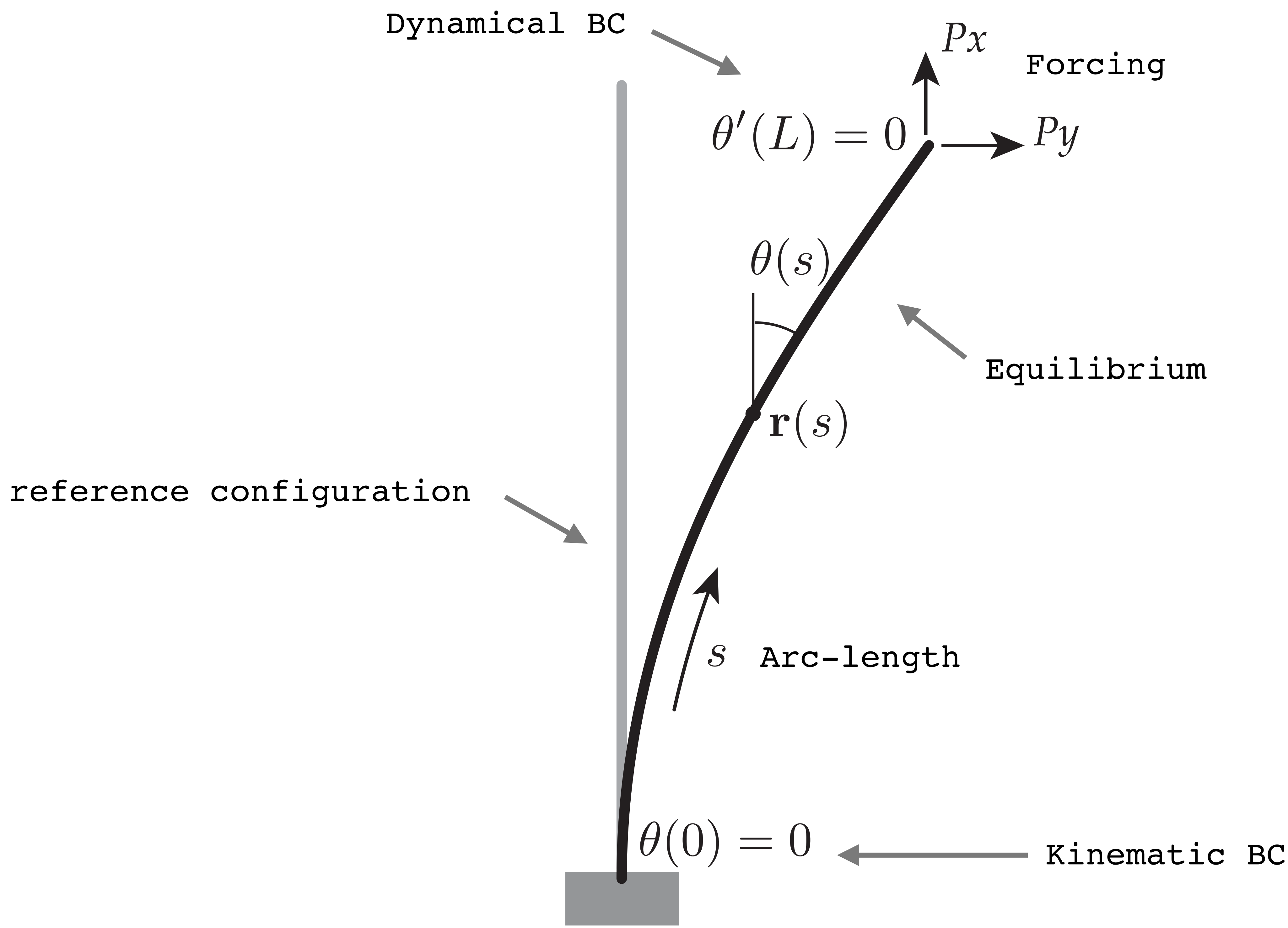
An introduction to Physical Mathematics



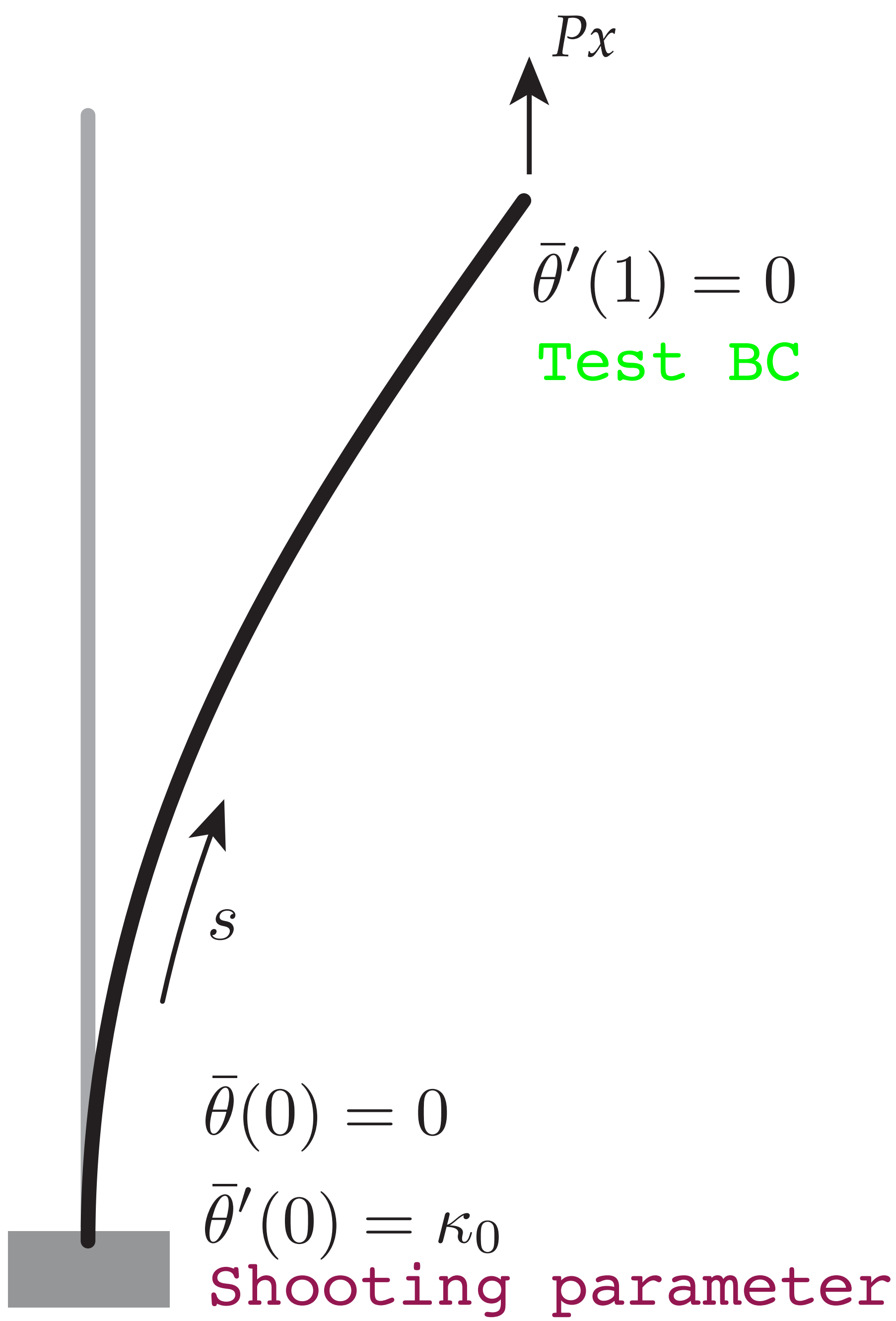
Euler's Elastica

$$\theta'' - \sin\theta = 0$$

Euler's Elastica



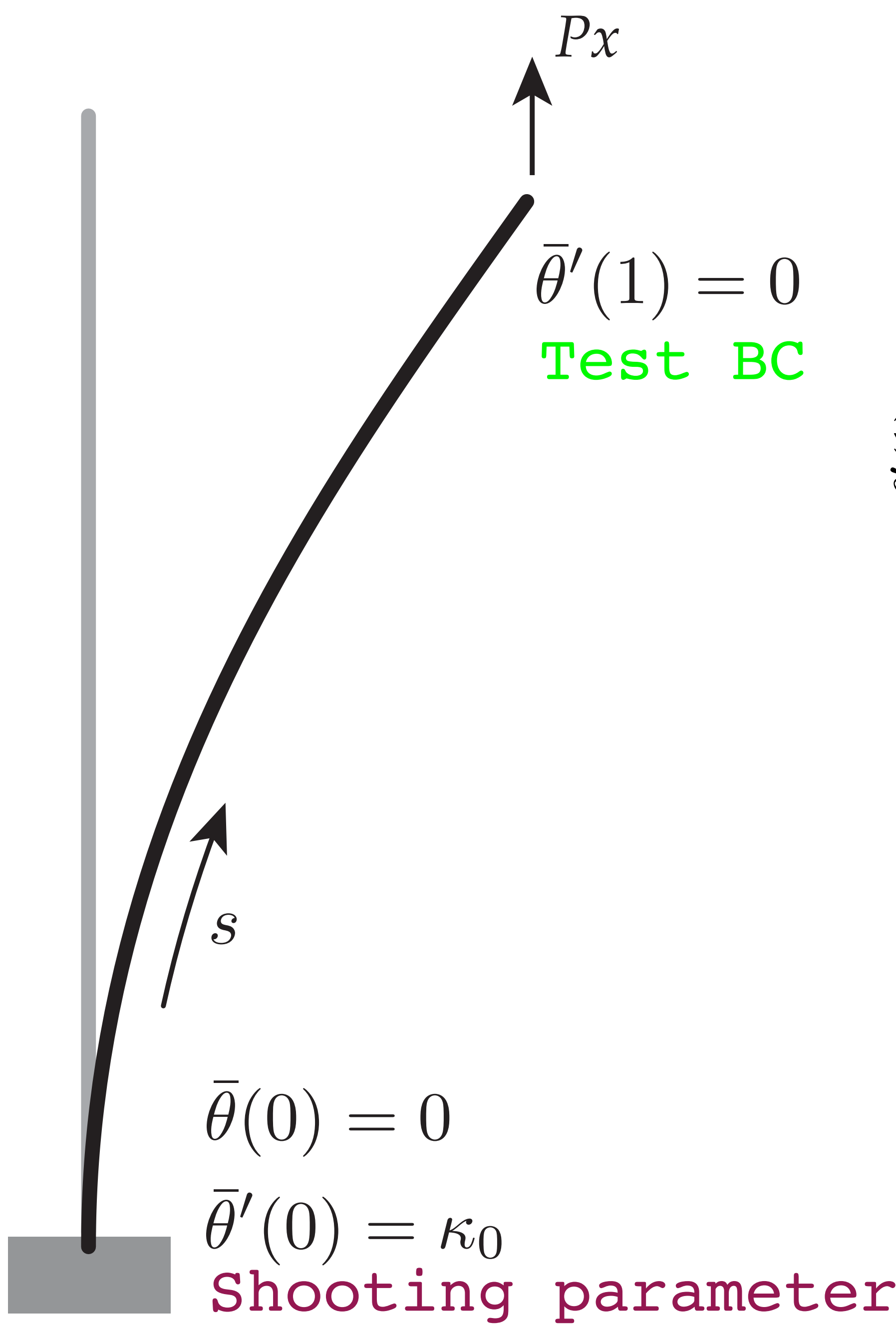
A shooting method



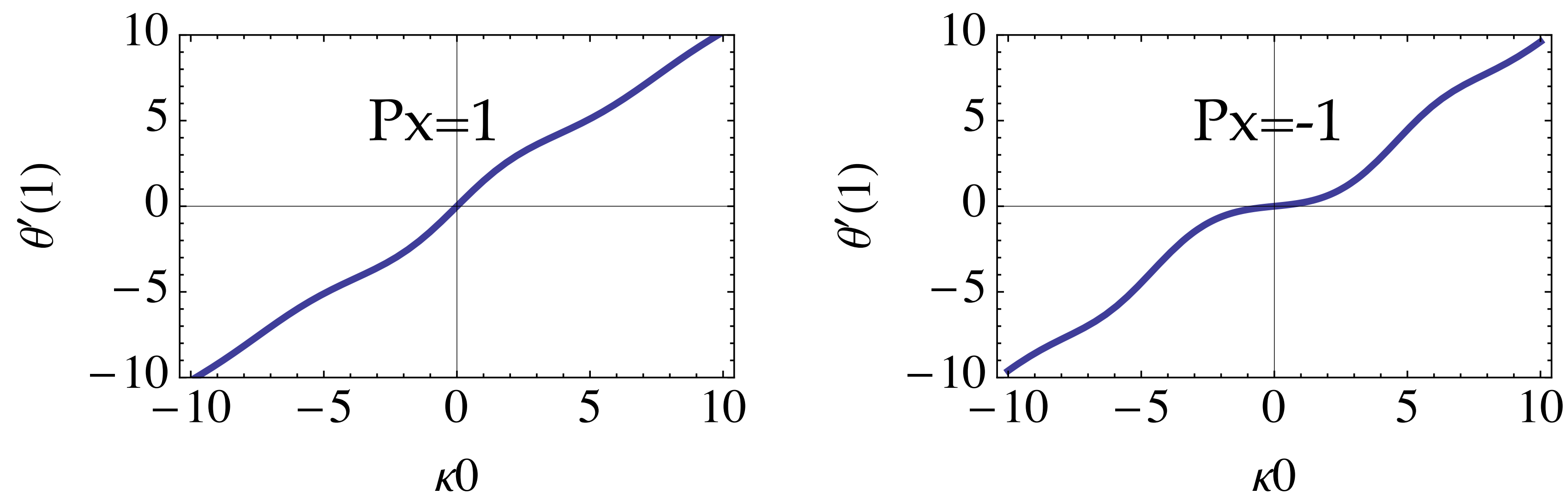
MMTK-Code

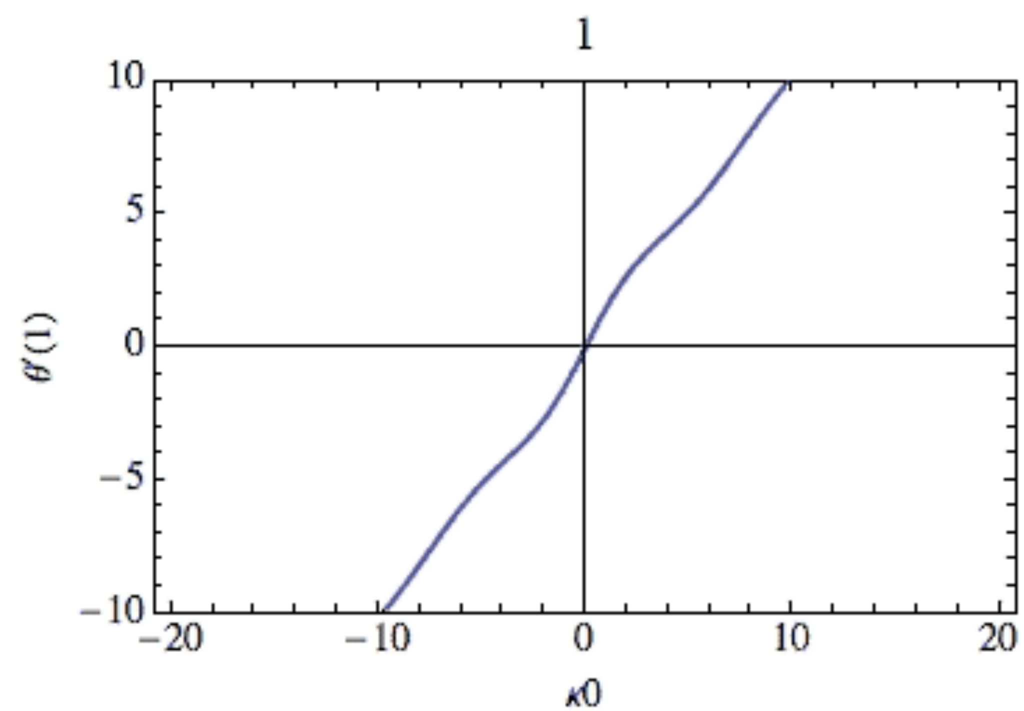
```
In[1]:= Eqsl = {
          θ''[s] - P Sin[θ[s]] == 0,
          x'[s] == Cos[θ[s]],
          y'[s] == Sin[θ[s]]
        }

Out[1]= {-P Sin[θ[s]] + θ''[s] == 0, x'[s] == Cos[θ[s]], y'[s] == Sin[θ[s]]}

In[2]:= Conds = {
           θ[0] == 0,
           θ'[0] == κ0,
           x[0] == 0,
           y[0] == 0
         };

In[3]:= sol = ParametricNDSolve[
          Join[Eqsl, Conds], {θ, x, y}, {s, 0, 1}, {κ0, P}
        ]

Out[3]= {θ → ParametricFunction[<>],
         x → ParametricFunction[<>], y → ParametricFunction[<>]}
```

A shooting method

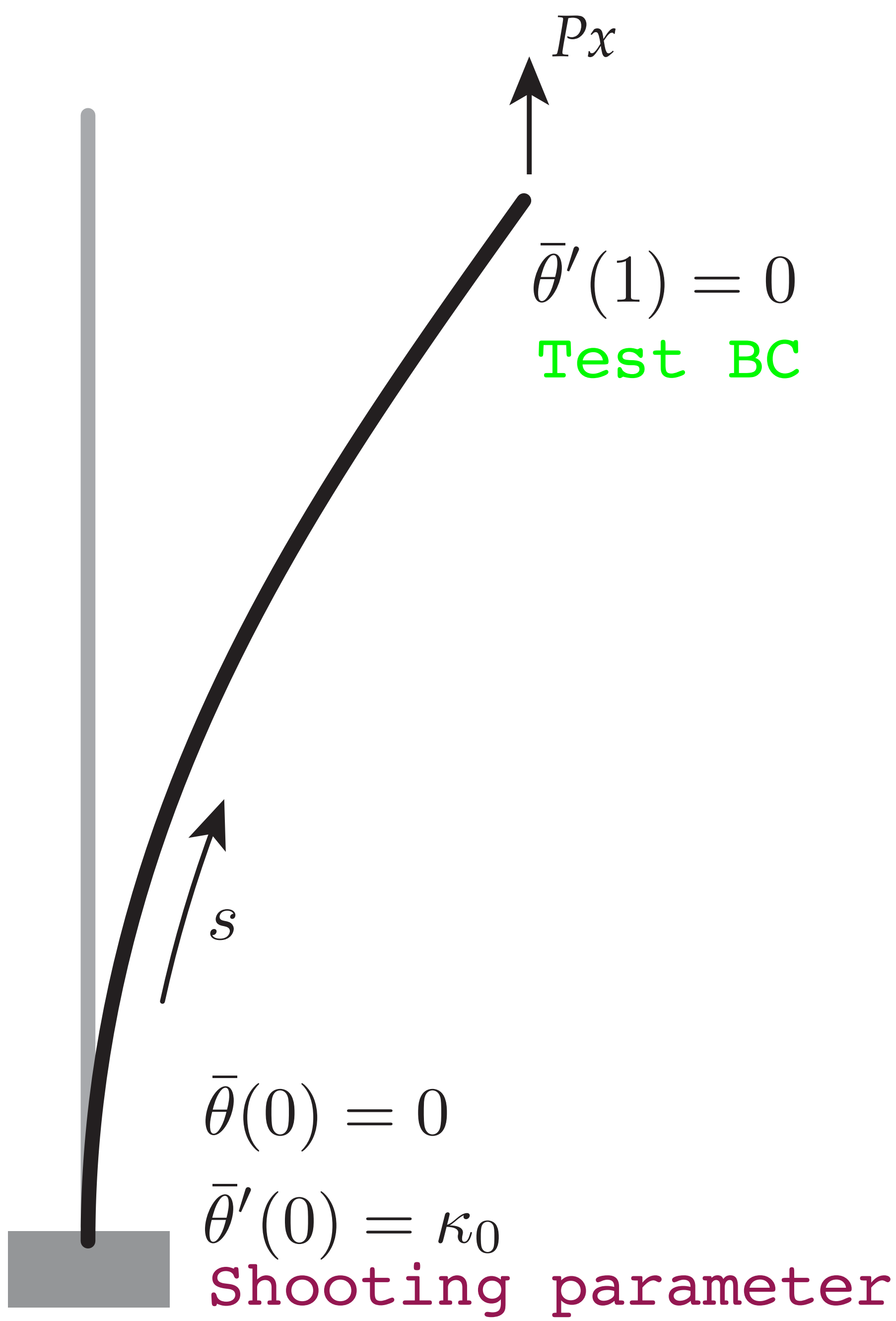


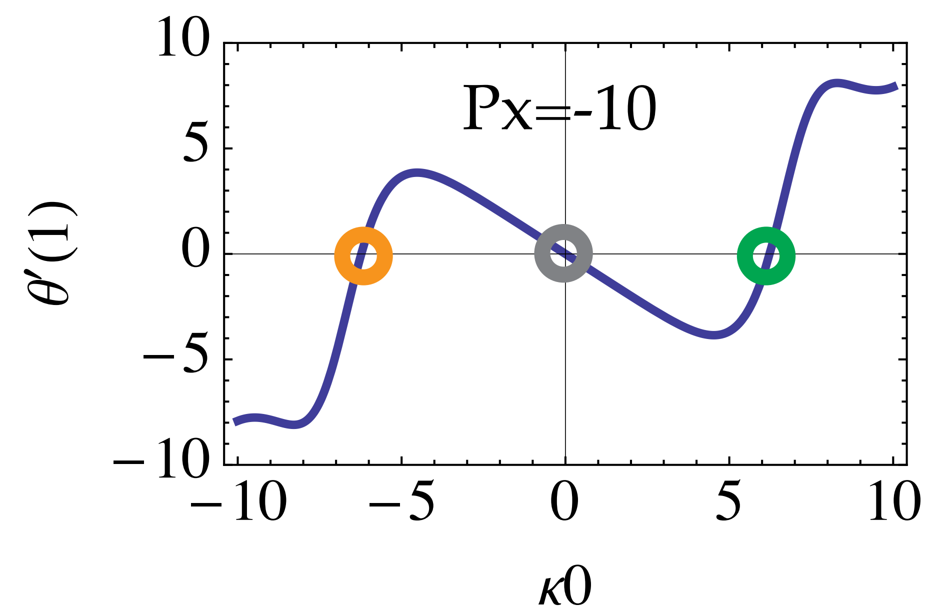

```
In[4]:= ΘPrime[κ0_, p_] := D[θ[κ0, p][s] /. sol, s] /. s → 1

In[5]:= Plot[ΘPrime[κ, -15], {κ, -10, 10}, PlotRange → {-10, 10}, Frame → True,
          PlotStyle → Thick, FrameLabel → {κ0, θ'[1]}, FrameStyle → Thickness[.003]]
```

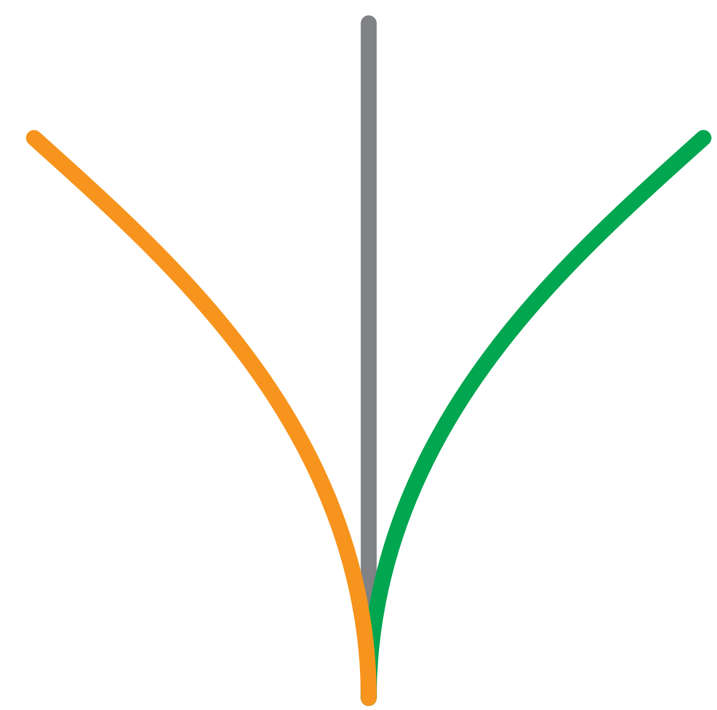




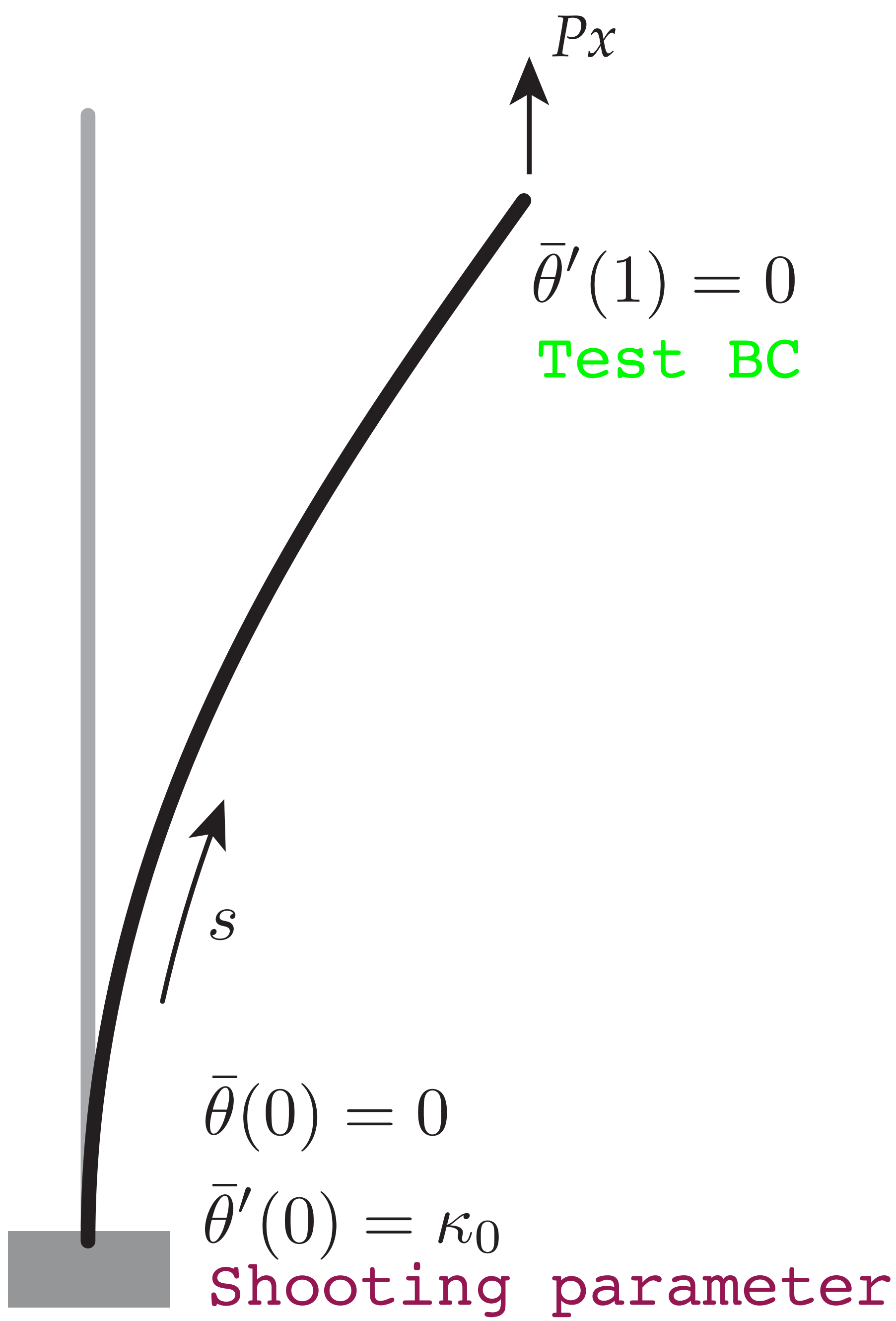
A shooting method





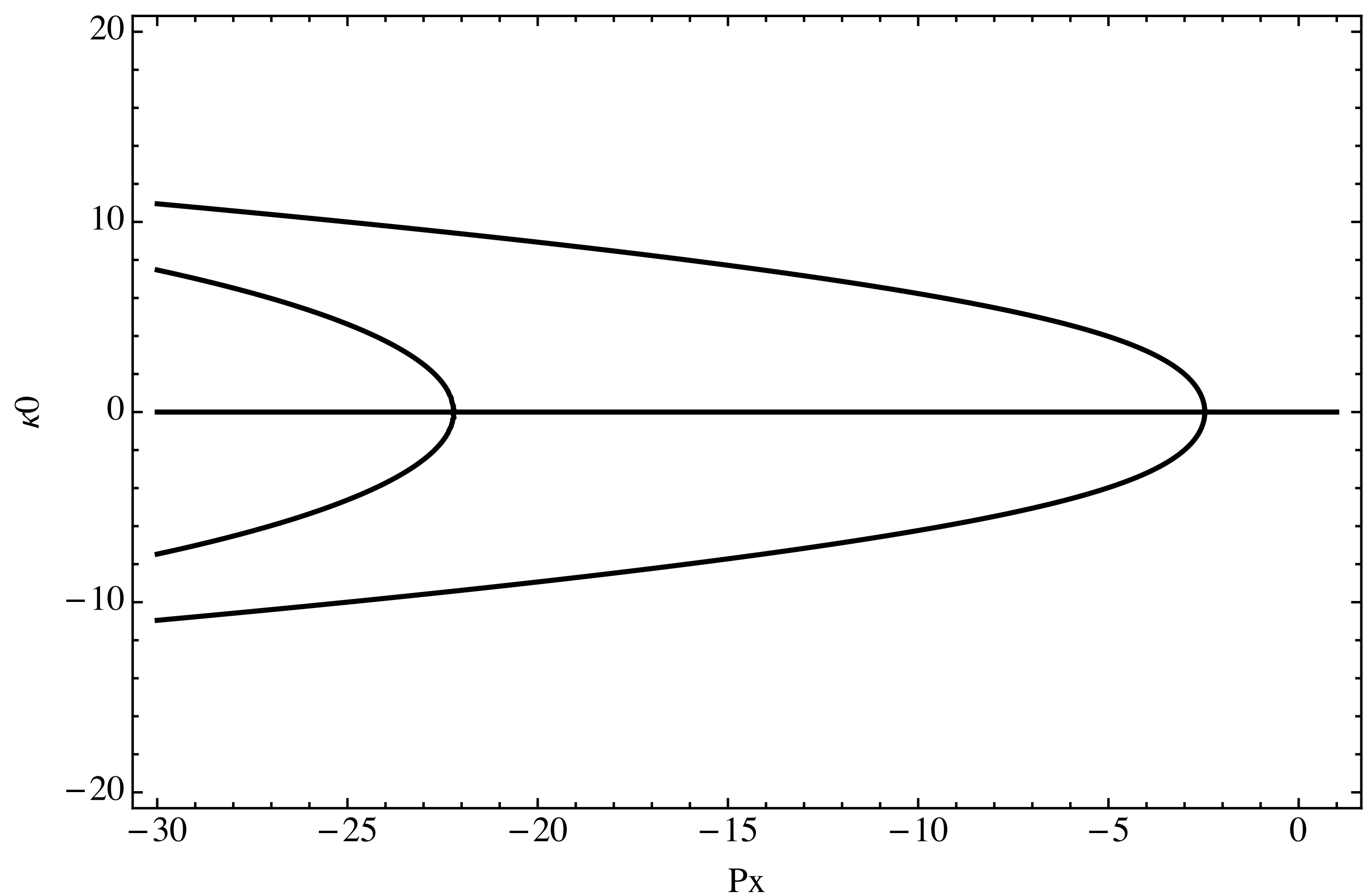


A shooting method

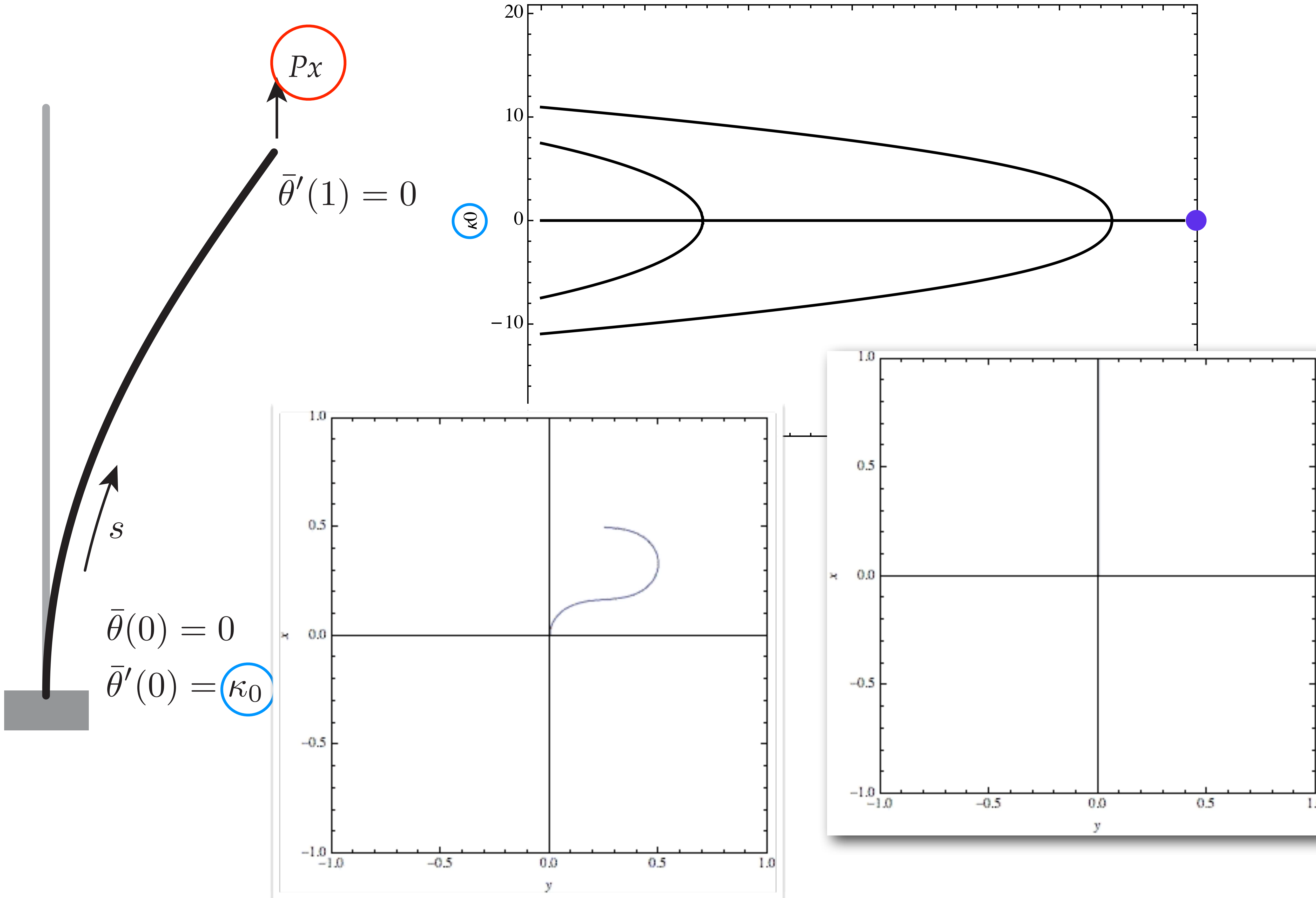


MMTK-Code

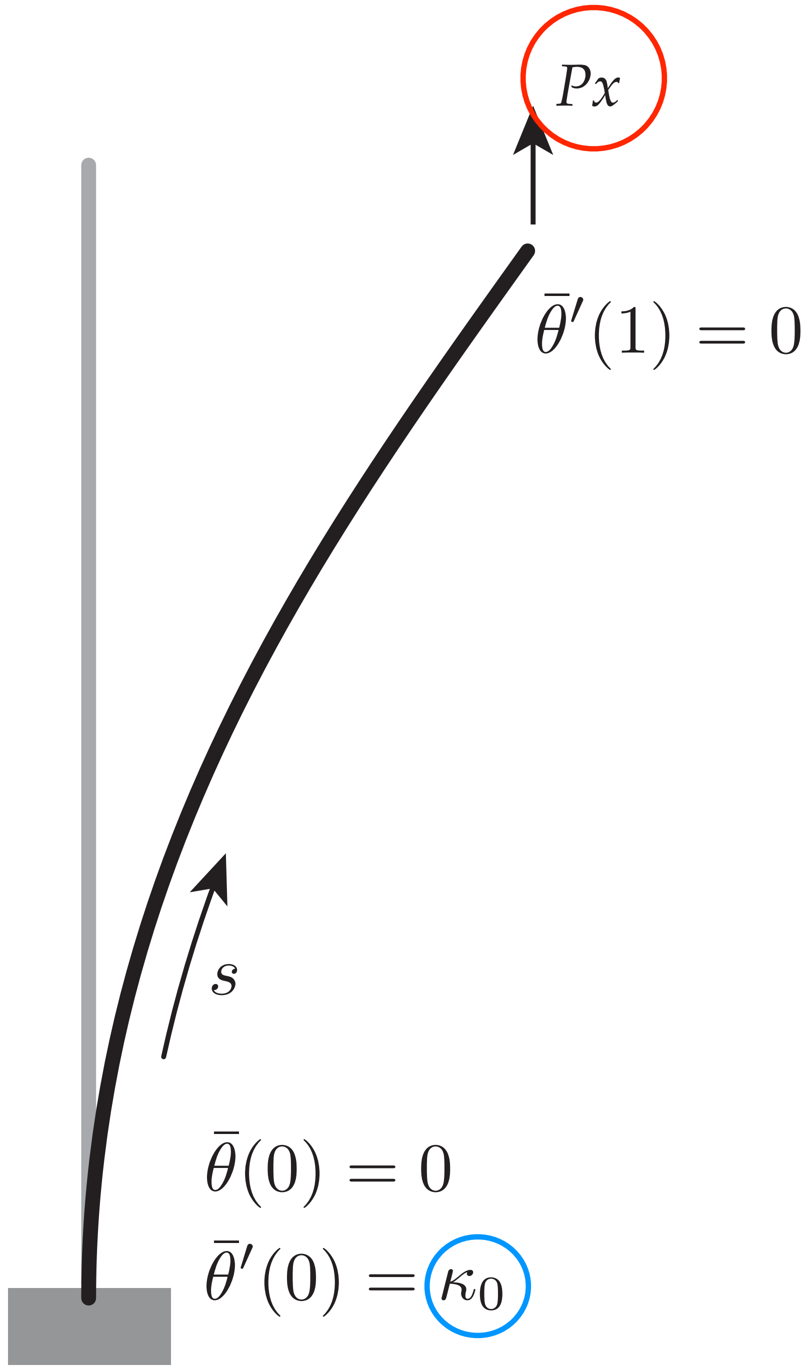
```
In[4]:= ΘPrime[κ0_, p_] := D[θ[κ0, p][s] /. sol, s] /. s → 1

plot = ContourPlot[ΘPrime[κ0, p] == 0, {p, -30, 1}, {κ0, -20, 20},
  MaxRecursion → 4, FrameLabel → {"Px", "κ0"}, ContourStyle → Thick,
  FrameStyle → Thickness[.002]]
```

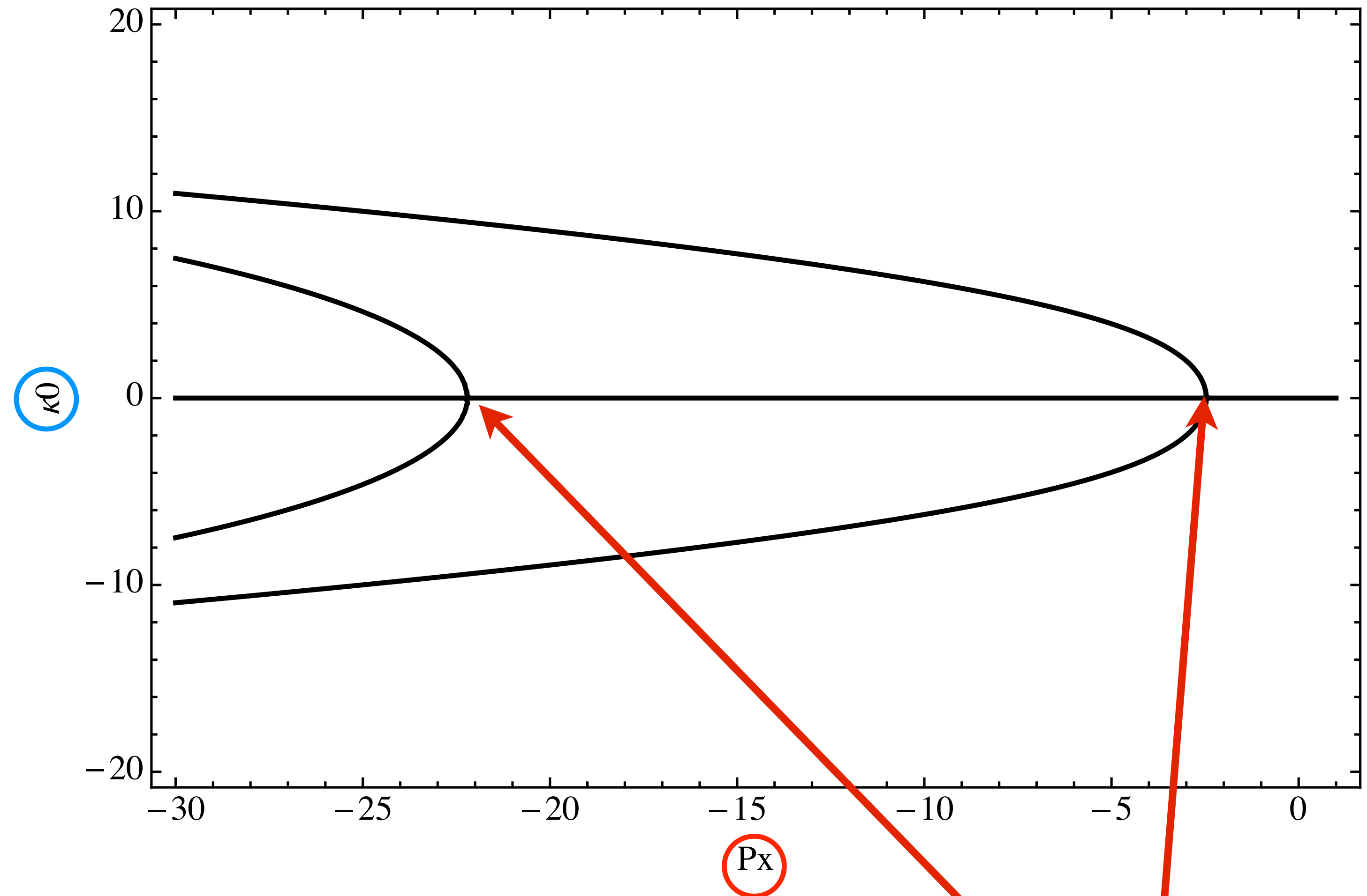


Branches of solutions



Branches of solutions





What about those bifurcation values?

$$P_{x,q} = -\left[\frac{(2q-1)\pi}{2}\right]^2$$