PROBLEM SET: CONVERGENCE OF MAJORITY DYNAMICS

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Abstract.

(1) A *dynamic monopoly* is a set $D \subset V$ in a graph $G = (V, E)$ with the following property: if $X_0^i = 1$ for all $i \in D$ and $X_0^j = 0$ for all $j \notin D$, then there exists a sequence of nodes $j_1, j_2, \ldots \notin D$ such that if node j_t updates in time t then then everyone eventually has opinion 1.
   (a) Give an example of a graph with 2000 nodes, where each node has degree 1000, and where there exists a dynamic monopoly of size 501.
   (b) Prove that in a graph with 200 nodes, where each node has degree 3, every dynamic monopoly has size at least 80.
   (c) Prove that there are no finite dynamic monopolies in infinite graphs.
(2) Convergence on $\mathbb{Z}^d$.
   (a) Calculate an upper bound on the maximal number of opinion changes in $\mathbb{Z}^d$, as a function of d.
   (b) Show that on $\mathbb{Z}^2$ the maximal number of opinion changes is exactly 2. This may require a direct combinatorial argument.
   (c) (Open problem, perhaps not hard). Calculate the maximal number of opinion changes on $\mathbb{Z}^3$.
(3) Prove the lemma from the lecture (namely that L_t is non-increasing).

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