CONVERGENCE OF MAJORITY DYNAMICS

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Abstract.

1. Graphs

(1) General graph (undirected).
(2) The FB graph.
(3) Degree of a node.

2. Majority dynamics

(1) Each person forms an initial binary opinion.
(2) Each microsecond a unique user (or no-one) updates (about 5000 updates on FB every second).
(3) Conformism: updates opinion to match majority, or leave unchanged if tied.

3. Convergence in finite graphs

(1) Theorem: regardless of the starting opinions and the order the updates are done in, the total number of changes of opinion is at most the number of edges divided by two.
(2) Proof.
    - Opinion $X_t^i \in \{0,1\}$.
    - $L_t = \sum_{\{i,j\} \in E} |X_t^i - X_t^j| \leq |E|$.
    - Majority dynamics means choosing the opinion that minimizes the number of disagreements with friends.
    - If anyone changes their opinion L goes down by at least two.

4. Infinite graphs

(1) Locally finite, countable graphs. Degree of a graph.
(2) Examples: $\mathbb{Z}, \mathbb{Z}^2, \mathbb{T}$.
(3) Convergence on $\mathbb{Z}$.
(4) Convergence on $\mathbb{Z}^2$? $\mathbb{Z}^3$?

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(5) No convergence on $\mathbb{T}$.

5. Convergence on infinite graphs

(1) The distance between two nodes in a graph is the length of a shortest path between them.
(2) Denote graph distance by $\Delta(\cdot,\cdot)$. Distance from node to edge.
(3) Let d be the smallest odd number larger than all the degrees.
(4) Let $a = (d-1)/(d+1)$.
(5) Fix $k \in V$. For $(i,j) \in E$, let $w(i,j) = a^{\Delta(k,(i,j))}$.
(6) Let $L_t = \sum_{(i,j)\in E} |X_t^i - X_t^j| w(i,j)$.
(7) Lemma: L_t is non-increasing.
(8) Theorem (T. & Tessler): The total number of changes of opinion k can make is

$$\sum_{(i,j)\in E} a^{\Delta(k,(i,j))}.$$

(9) Convergence in $\mathbb{Z}^2$.