Math 18.02, r11 Problems #14 June 30, 2013

- 1. Sketch the following vector fields:
  - (a)  $\vec{F}_1(x,y) = y\hat{\imath} + \hat{\jmath}$
  - (b)  $\vec{F}_2(x, y) = y \cos x \,\hat{\imath} + \sin x \,\hat{\jmath}$
  - (c)  $\vec{F}_3(x,y) = -\frac{y}{\sqrt{x^2+y^2}}\hat{\imath} + \frac{x}{\sqrt{x^2+y^2}}\hat{\jmath}$
- 2. Give the equation defining each of the vector fields illustrated below:



- 3. (4B-2) Evaluate the integral of  $\vec{F} = x\hat{i} + y\hat{j}$  along the path C which goes once counterclockwise around a circle of radius a centered at the origin. First argue geometrically, and then check your answer by directly computing the integral.
- 4. For the first and second vector fields of problem 1, compute the integral along three different paths from (0,0) to (1,1):
  - (a)  $C_1$ , a straight line from (0,0) to (1,1)
  - (b)  $C_2$ , a line from (0,0) to (1,0) and then to (1,1).
  - (c)  $C_3$ , along the parabola  $y = x^2$
- 5. One of the vector fields from the previous question is conservative. Which one? Find a function f(x, y) of which it is the gradient, and evaluate the above integral using the fundamental theorem for line integrals.
- 6. What is the gradient field associated with the function  $f(r, \theta) = r \log r$ ?

