

18.089 REVIEW OF MATHEMATICS

HOMEWORK 0

exercise 1. Compute the derivatives of the following functions:

- $y(x) = x^3$ (using the definition).
- $y(x) = x^n$ (using the definition).
- $y(x) = \frac{1}{\sqrt{x}}$.
- $y(x) = \frac{1}{\sqrt{x^2-1}}$.
- $y(x) = (x^3 + 1)(x^2 - 2x)$.
- $y(x) = f(x)g(x)h(x)$.
- $y(x) = \frac{\sqrt{x}}{x^2-1}$.
- $y(x) = \left(\frac{1}{x^2+1}\right)^2$.
- $y(x) = (x^4 + 3x)^3$.

exercise 2. Study the following functions (i.e. graph them and determine the nature of the critical points):

- $y = 3x^2 + 1$.
- $y = x^2 - 3x + 2$.
- $y = x^3$.
- $y = x^3 + 1$.

exercise 3. Compute the tangent lines of the above functions at the points $(0, 1)$, $(1, 0)$, $(0, 0)$ and $(1, 2)$ respectively.

exercise 4. Compute the tangent line of the function $y = \sqrt{x^3 + 3x}$ at the point $(1, 2)$.

exercise 5. Compute the tangent lines of the curve $x^2 + y^2 = 25$ at the points $(3, 4)$ and $(4, 3)$. Where do these lines intersect?

exercise 6. Minimize the perimeter of a rectangle with area= 20.

exercise 7. A ball travels on the parabola of equation $y - x^2 = 0$. At each time t , denote by $x(t)$ and $y(t)$ the projections of the ball on the x and y-axis respectively. If you know that the speed of $x(t)$ is constant and equal to 3, what is the speed of $y(t)$ when $x(t) = 1$ (and hence $y(t) = 1$)?

exercise 8. Use trigonometric formulas and implicit differentiation to show that $(\cos^{-1})' = \frac{-1}{\sqrt{1-x^2}}$.

exercise 9. Compute the following integrals:

- $\int \frac{x^3}{x^4+5} dx$.
- $\int \theta \sin(\theta^2) d\theta$.
- $\int x \exp^{x^2} dx$.
- $\int \frac{3x}{\sqrt{x^2+1}} dx$.

exercise 10. Compute the following integrals using substitutions:

- $\int e^x (e^x + 1)^b dx$, $u = e^x + 1$.
- $\int \frac{dx}{\sqrt{9-4x^2}} = \frac{1}{3} \int \frac{dx}{\sqrt{1-(\frac{2x}{3})^2}}$.
- $\int \frac{2x+1}{x^2+x+1} dx$.
- $\int \frac{x}{\sqrt{x^2+1}} dx$.
- $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$.

exercise 11. Compute the following integral: $\int \frac{dx}{x^2+6x+25}$.

exercise 12. Find the area between the curves $y = x^2 + 2$ and $y = 4 - x^2$.

exercise 13. Find the volume of the solid obtained by revolving around the x-axis the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$.

exercise 14. Calculate the following integrals:

- $\int \frac{x+2}{(x-3)^2(x+1)} dx$.
- $\int \frac{x+2}{(x-6)(x+5)} dx$.
- $\int \frac{x^3}{(x-2)(x+2)} dx$ (note that this is not a proper fraction).

exercise 15. Compute • $\int_1^{\sqrt{e}} x^3 \log 2x dx$. • $\int e^{3x} \cos 2x dx$. • $\int (x^2 + 1) \cos 3x dx$.

exercise 16. Compute the area inside the circle $x^2 + y^2 = 4$ and the parabola $y = x^2$.