18.089 Exam 2

July 5, 2013

Name:

This exam consists of nine problems, not arranged in any particular order. Please solve all problems in the space provided (or attaching additional sheets as necessary), showing all work as neatly and cleanly as possible.

All work must be your own. In particular, you may not seek help from other students in the class or any "live" internet resources. But feel free to use reference works such as your class notes, a textbook, or Wikipedia. If you have any questions about what constitutes an acceptable source, please ask.

Problem	**Value**	**Score**
Problem 1	15	
Problem 2	20	
Problem 3	30	
Problem 4	15	
Problem 5	30	
Problem 6	20	
Problem 7	20	
Problem 8	25	
Problem 9	25	
Total	200	

Problem 1 (*15 points*)**.** Compute the first five terms (i.e., up to and including the x^4 term) of the Taylor series for $f(x) = e^{2x} \cdot \cos x + \sin(3x)$. (Hint: there are two natural ways to proceed – either by taking derivatives or by using some Taylor series you already know.)

Problem 2 *(10+10 point)*. (a) Determine the angles of the triangle with vertices $P_0 = (1, -4, 1)$, $P_1 = (1, 1, 1)$ and $P_2 = (4, 1, 5)$.

(b) Determine the implicit equation of the plane containing the above triangle.

Problem 3 (*10+10+10 points*)**.** A particle travels in space so that its position at time t is given by the parametric equation

$$R(t) = (t \sin t, t \cos t, t).$$

(a) Determine the velocity, the acceleration and the speed of the particle at the point $R(0)$.

(b) Write down (but do not evaluate) the integral that gives the arclength of the portion of the curve between the points $R(0)$ and $R(\pi/2)$.

(c) Show that the curve lies on the surface $x^2 + y^2 - z^2 = 0$ and compute the tangent plane to this surface at $R(\pi/2)$.

Problem 4 (*10+5 points*)**.** Consider the curve given by the polar equation $r = 1 - 2\cos\theta$.
(a) Compute the area bounded by this curve (be careful with the limits of integration!).

(b) Write down (but do not evaluate) an integral whose value is the arclength of this curve.

Problem 5 (*10+10+10 points*)**.** Let $z = f(x,y) = e^{x+y} + e^{xy+1}$.
(a) Find the equation of the tangent plane to the surface at the point $(2, 1, 2e^3)$.

(b) Find the implicit and explicit equation of the line perpendicular to the plane passing through $(2, 1, 2e^3)$.

(c) Compute the directional derivative at the point $(1, -1)$ in the direction $\vec{v} = (3, -3)$.

Problem 6 (*10+10 points*)**.** The plane given by $x+y+2z=2$ and the paraboloid $z=x^2+y^2$ intersect in an ellipse.
(a) Find the points on this ellipse that are nearest to and farthest from the origin.

(b) Compute the tangent lines to the ellipse at those points.

Problem 7 (*10+10 points*)**.** (a) Find the critical point of the function $f(x, y) = x^2 + xy + y^2 - 4x - 5y + 5$ and classify it.

(b) For $x = t - s$, $y = t + s$, compute $\frac{\partial f}{\partial t}$ and $\frac{\partial f}{\partial s}$ and check that $\frac{\partial f}{\partial s} + \frac{\partial f}{\partial t} = 2\frac{\partial f}{\partial y}(t - s, t + s)$.

Problem 8 (*10+5+5+5 points*)**.** This problem deals with the double integral

$$\int_{y=0}^{2} \int_{x=y}^{2} xy\,dx\,dy.$$

(a) Draw the region R over which the integral is being taken, and evaluate the double integral directly.

(b) Evaluate the integral by changing the order of integration, so that the inside integral is taken with respect to y.

(c) Evaluate the integral by switching to polar coordinates.

(d) Evaluate the integral using the change of coordinates $u = y/x$, $v = x$.

Problem 9 (*5+5+10+5 points*). Give bounds on integration for the following two- and three-dimensional regions, in the coordinate systems indicated.

(a) (Rectangular) The parallelogram with vertices at $(0, 0)$, $(1, 0)$, $(1, 1)$, $(2, 1)$.

(b) (Polar) A circle centered at the point $(1, 0)$, with radius 1.

(c) (Cylindrical, spherical) A cone with vertex at $(0, 0, 0)$ whose base is a circle of radius 2 centered at $(0, 0, 2)$ (parallel to the xy-plane).

(d) Set up integrals to compute the z-coordinate of the center of mass of the cone from (c), assuming a uniform density $\delta(x, y, z) = 1$. You don't have to actually compute it.