18.089 REVIEW OF MATHEMATICS

HOMEWORK 3, DUE ON FRIDAY, JULY 5

Solve as many problems as you want. Only problems labeled with a $\star$ are required.

Thursday, June 27.

Exercise 1. Compute dw/dt for $w = ln(x^4 + 2x^2y + 3y^2)$, $x(t) = t$, $y(t) = 2t^2$.

Exercise 2 ($\star$)**.** For $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, show that

$$(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 = (\frac{\partial z}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial z}{\partial \theta})^2.$$

Exercise 3 ($\star$)**.** For the following functions find the critical points and classify them.
• $z = x^2 + y^3 - 6xy$. • $z = xy(2x + 4y + 1)$.

Exercise 4 ($\star$)**.** If the sum of three numbers x, y and z is 12, what must these numbers be for the product of x, y^2 and z^3 to be as large as possible?

Friday, June 28.

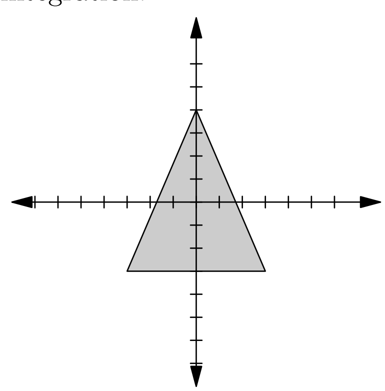
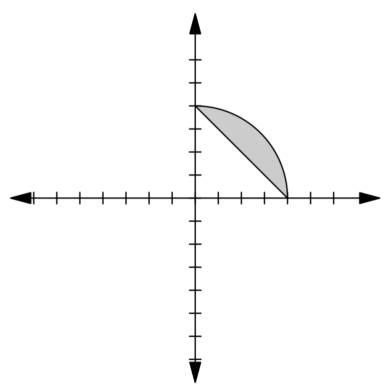
Exercise 5 ($\star$)**.** Find the rectangle of maximum area (with sides parallel to the axes) that can be inscribed in the ellipse $x^2 + 4y^2 = 4$.

Exercise 6. Find the maximum and minimum value of $f(x, y) = x^2 - xy + y^2$ on the circle $x^2 + y^2 = 1$.

Exercise 7 ($\star$)**.** Find the maximum value of $f(x, y, z) = 2x + 2y - z$ on the sphere $x^2 + y^2 + z^2 = 1$.

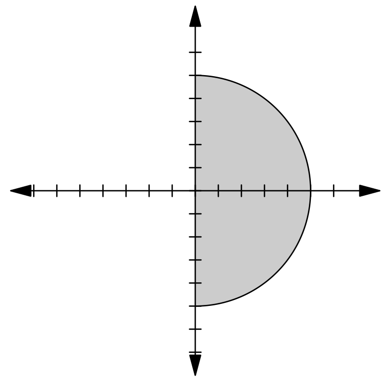
Monday, July 1.

Exercise 8 ($\star$)**.** Write down bounds for integration over the following regions. Try to give both possible orders of integration.

Exercise 9. Compute $\iint_R f(x, y)\, dA$, where R is the rectangle $-2 \le x \le 2$, $-1 \le y \le 1$ and $f(x, y) = xe^{xy}$. One of the possible orders of integration will be easier – which one?

Exercise 10 ($\star$)**.** Find the center of mass of the following semicircular region, assuming the density is a uniform $\delta(x, y) = 1$. The center of the circle is $(0, 0)$, and the radius is 5.



Tuesday, July 2.

Exercise 11 ($\star$)**.** Compute

$$\int_{x=-\infty}^{\infty} \int_{y=1}^{2} \frac{1}{(x^2+y^2)^2}\, dy\, dx$$

by switching to polar coordinates. Hint: the trig identity $\sin^2 t = \frac{1-\cos 2t}{2}$ just might come in handy.

Exercise 12. Find the center of mass of the northern hemisphere, assuming that the earth is a sphere of radius $4,000$ miles and uniform density (not a very good assumption), centered at $(0,0,0)$. What line of latitude has the same z-coordinate as this point?

Exercise 13 ($\star$)**.** Find the volume of the region in space bounded on the sides by the xz- and yz-planes, below by the plane $x+y+z=1$, and above by the plane $x+y+z/2=1$. It will be helpful to sketch this region first!

Wednesday, July 3.

Exercise 14 ($\star$)**.** Let R be a parallelgram with vertices at $(0,0)$, $(1,2)$, $(2,1)$, and $(3,3)$. Compute

$$\iint_R 3x\, dx\, dy$$

using an appropriate change of coordinates.

Exercise 15 ($\star$)**.** Give bounds for triple integrals over the following regions, using the specified coordinates:

- Cylindrical: a cone whose base is the unit circle in the xy-plane, with the point of the cone at $(0,0,3)$.
- Spherical: the first octant of a sphere of radius 1 centered at the origin (i.e. the eighth of the sphere where x, y, and z are all positive).
- Both: the region between the planes $z=1$ and $z=2$.