18.089 REVIEW OF MATHEMATICS

HOMEWORK 2, DUE ON FRIDAY, JUNE 28

Solve as many problems as you want. Only problems labeled with a $\star$ are required.

Friday, June 21.

Exercise 1 ($\star$)**.** Find the velocity, the acceleration and the tangential and normal components of the acceleration in the following cases:

$$\bullet\ (2t-5, t^3+3) \quad \bullet\ (t\cos t, t\sin t).$$

Exercise 2. Sketch the graph of the following curves by finding the equations in terms of x and y:

$$\bullet r = 6\sin\theta \qquad \bullet\ r = 4(\sin\theta + \cos\theta).$$

Exercise 3. Write the following curves in polar coordinates:

$$\bullet x^2+y^2=9 \quad \bullet\ x^2-6x+y^2=9 \quad \bullet\ y=x^2.$$

Exercise 4 ($\star$)**.** Recall that

$$\begin{cases} \cos^2\theta = \frac{\cos 2\theta+1}{2} \\ \sin^2\theta = \frac{1-\cos 2\theta}{2}. \end{cases}$$

Find the area inside the curve $r = (1+\cos\theta)$.

Exercise 5. Find the area inside the curve $r = \sin 2\theta$ (note that the limit of integration is NOT $[0, 2\pi)$).

Exercise 6 ($\star$)**.** Find the area of the region inside the curve $r = 2a\cos\theta$ and below the line $x = y$.

Monday, June 24.

Exercise 7. Determine and draw the domain of the function $z = \frac{1}{4x^2-y^2}$.

Exercise 8. Find the partial derivatives of $z = y\cos x$ and $z = x^2\sin y$.

Exercise 9 ($\star$)**.** Consider the surface $z = 2x^2+y^2$.

a) The plane $y = 3$ intersects the surface in a curve. Find the equation of the tangent line to this curve at $x = 2$.

b) The plane $x = 2$ intersect the surface in a curve. Find the equation of the tangent line to this curve at $y = 3$.

Exercise 10 ($\star$)**.** Compute the tangent plane to the following surfaces at the points indicated.

a) $z = \exp y\cos x$ at $(0, 0, 1)$.

a) $xy^2+yz^2+zx^2 = 25$ at $(1, 2, 3)$.

Tuesday, June 25.

Exercise 11 ($\star$)**.** Determine wether or not the following functions are continuous and differentiable or just continuous at $(0,0)$ (or not even continuous). In case they are differentiable, compute dz and the tangent plane at $(0,0)$.
$z = \begin{cases} \frac{2xy}{(x^2+y^2)^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$, $z = \sin\sqrt{x^2+y^2}$.

Exercise 12 ($\star$)**.** Compute the gradient, evaluate it at the point P and compute the directional derivative at P in the direction $\vec{v}$ of the following functions:

(a) $f(x,y) = x^2y + xy^2$, $P = (-1,2)$, $\vec{v} = (3,-4)$.
(b) $g(x,y) = \sqrt{x^2+y^2}$, $P = (2,6)$, $\vec{v} = (1,1)$.

Wednesday, June 26.

Exercise 13. Compute the gradient, evaluate it at the point P and compute the directional derivative at P in the direction $\vec{v}$ of the following functions:
(a) $g(x,y,w) = \sqrt{x^2+y^2+z^2}$, $P = (2,6,-3)$, $\vec{v} = (1,1,1)$.
(b) $h(w,x,y,z) = wx + wy + wz + xy + xz + yz$, $P = (2,0,-1,-1)$, $\vec{v} = (1,-1,1,-1)$.

Exercise 14 ($\star$)**.** Find the maximum value of the directional derivative of f at P and the direction in which it occurs :
(a) $f(x,y,z) = \sin xy + \cos yz$, $P = (-3, 0-7)$.
(b) $f(x,y,z) = e^x \cos y + e^y \cos z + e^z \cos x$, $P = (0,0,0)$.

Exercise 15. Find the normal line to the curve $xy^2 - 2x^2y = -1$ at the point $(1,1)$.

Exercise 16 ($\star$)**.** Find the tangent plane to the surface $xy^2z^3 = 12$ at the point $(3,-2,1)$.