## 18.089 REVIEW OF MATHEMATICS

HOMEWORK 1, DUE ON FRIDAY, JUNE 20

Only the problems labeled with a $\star$ are required (and these are the ones that will determine your homework grade). You may turn in as many additional problems as you want: all will be corrected.

**Monday, June 17**

**Exercise 1.** Let $\vec{v} = (1, 2, -1)$ and $\vec{v}' = (1, 1, 0)$. Find a vector $\vec{w}$ such that $\vec{v} \perp \vec{w}$, $\vec{v}' \perp \vec{w}$ and $|\vec{w}| = \sqrt{3}$.

**Exercise 2 ($\star$).** Using the dot product (as we did in class), find the area of the parallelogram spanned by $\vec{v} = (2, 3, 4)$ and $\vec{w} = (1, 1, 1)$. Do the same using cross product.

**Exercise 3 ($\star$).** Find the angle $\theta$ between the diagonal of a cube and:

a) an adjacent edge.
b) an adjacent diagonal of a face.

**Exercise 4 ($\star$).** We know that for any two vectors $\vec{v} = (a, b, c)$ and $\vec{w} = (d, e, f)$ and any number $\lambda$, $(\vec{v} \times \vec{w}) = (g, h, i)$ is perpendicular to both $\lambda\vec{v}$ and $\lambda\vec{w}$. Using this fact prove that

$$det \begin{pmatrix} \lambda a & \lambda b & \lambda c \\ a & b & c \\ d & e & f \end{pmatrix} = 0$$

**Exercise 5.** Use the dot product to show that the triangle with vertices $P = (2, 7, -2)$, $Q = (0, 4, -1)$ and $R = (1, 4, 1)$ is a right triangle.

**Exercise 6.** Calculate $\vec{v} \times \vec{w}$ and check that $\vec{v} \times \vec{w} \perp \vec{v}, \vec{w}$ in the following cases:

- $\vec{v} = (2, 2, -1)$, $\vec{w} = (1, 1, 1)$.
- $\vec{v} = (5, -4, 3)$, $\vec{w} = (-3, -2, 1)$.

**Tuesday, June 18.**

**Exercise 7.** Let $A$ be the matrix $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$. Find $\vec{v}$ such that $A \cdot \vec{v} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$.

**Exercise 8 ($\star$).** Let $A$ be the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & -2 & 4 \end{pmatrix}$. Determine whether or not $A$ is invertible. If it is, find $A^{-1}$.

**Exercise 9.** • Find both the parametric equation and the implicit equation of the line containing $P_0 = (1, -1, 1)$ and with direction $\vec{v} = (1, 1, 1)$.
• Find the plane perpendicular to the above line containing $P_0$ (hint: $\vec{v}$ is perpendicular to the plane).

**Exercise 10 ($\star$).** a) Find the parametric equation of the line through $P_0 = (1, -1, 0)$ and $P_1 = (2, 1, -2)$.

b) Find the plane through the points $P_2 = (1,0,0)$, $P_3 = (2,1,0)$ and $P_4 = (-1,0,1)$; give both a parametric and an implicit equation for it. Write down a normal vector to the place, and check that it is perpendicular to the vectors $\vec{v}$ from $P_2$ to $P_3$ and $\vec{w}$ from $P_2$ to $P_4$.
c) Find the intersection of the plane in b) with the line from a).

**Exercise 11** ($\star$)**.** Find the plane containing the intersection of $x-2y-5z-3=0$ and $5x+y-z-1=0$ and parallel to the plane $4x+3y+4z+7=0$. (Hint: the normal vector to a plane is the same as the normal vector to a plane parallel to it.)

**Exercise 12.** Show that the two lines

$$\begin{cases} x = 1+t \\ y = 2t \\ z = 1+3t \end{cases} \qquad \begin{cases} x = 3s \\ y = 2s \\ z = 2+s \end{cases}$$

intersect. Find the point of intersection.

**Wednesday, June 19.**

**Exercise 13** ($\star$)**.** Use the comparison or limit comparison test to say whether the following sums converge:

$$\sum_{n=2}^{\infty} \frac{1}{\ln n}; \qquad \sum_{n=1}^{\infty} \frac{\ln n}{n^3}; \qquad \sum_{n=2}^{\infty} \frac{2n}{n^2+1}.$$

**Exercise 14.** Use the integral test to say whether the following series converge. (Note thst the unusual initial values in the second and third examples are just there so that the series really are decreasing and positive.)

$$\sum_{n=1}^{\infty} \frac{\operatorname{actan} n}{1+n^2}; \qquad \sum_{n=3}^{\infty} \frac{1}{n\cdot(\ln n)\cdot \ln(\ln n)}; \qquad \sum_{n=3}^{\infty} \frac{1}{n\cdot(\ln n)\cdot(\ln(\ln n))^2}.$$

**Exercise 15** ($\star$)**.** Use the root or ratio test to say whether the following series converge.

$$\sum_{n=1}^{\infty} \frac{n!}{(2n)!}; \qquad \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}; \qquad \sum_{n=1}^{\infty} \frac{2^{3n}}{3^{2n}}.$$

**Exercise 16** ($\star$)**.** Compute the first four non-zero terms of the power series of the following functions, using a method of your choice.

$$\frac{1}{1+x}; \qquad e^{x^2}; \qquad \sin^2 x; \qquad \frac{1}{1+x^2}.$$

**Thursday, June 20.**

**Exercise 17.** Find the implicit equation of the curve $R(t) = (\cos^2 t, \sin^2 t)$ and sketch it. Note that both $x(t)$ and $y(t)$ has to be positive.

**Exercise 18** ($\star$)**.** Find the implicit equation for the curve $R(t) = (t^3, 1-t^2)$. Sketch the curve using the fact that you already know the sketch of $(t^2, t^3)$. Find the velocity vector and the speed of it and find a vector that is perpendicular to the tangent to the curve at the point $(1,0)$ (note that $(1,0) = R(1)$).

**Exercise 19.** Find the equation of the line tangent to $R(t) = (\cos t, \sin t, t)$ at the point $(1/2, \sqrt{3}/2, \pi/3)$.

**Exercise 20** ($\star$)**.** Compute the velocity vector $\vec{v}(t)$, the speed and the acceleration $\vec{a}(t)$ of the curve $R(t) = (t^2 - 1, 3t^2, \ln t)$. Find the implicit equation of the plane through $R(1)$ with direction vectors $\vec{v}(1)$ and $\vec{w} = (1, 3, 1)$.

**Exercise 21.** Compute the velocity vector $\vec{v}(t)$, the speed, the unit tangent vector $\vec{T}$ and the acceleration $\vec{a}(t)$ of the ellipse $R(t) = (4\cos 2t, 3\sin 2t)$. Is $\vec{v}(t) \perp \vec{a}(t)$?

**Exercise 22.** Compute the curvature $k$ in the following two cases:
- $y = x + \frac{1}{x}$.
- $R(t) = (\exp t \sin t, \exp t \cos t)$.

(Questions on material from Friday, June 21st will be on the next problem set!)