parametrized surface, again.

"$u=\theta$"

"$v=\varphi$"

that Stupid ellipsoid.

$0 \le u \le 2\pi$

$0 \le v \le \pi/2$

~~$z = c\cos v$~~

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}.$$

$x(u,v) = a \sin v \cos u$

$y(u,v) = b \sin v \sin u.$

$z(u,v) = c \cos v.$

$\vec{F}(x,y,z) = z\hat{k}.$

$$\vec{dS} = \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\, du\, dv = (-a \sin v \sin u,\ b \sin v \cos u,\ 0)$$

$$\times (a \cos v \cos u,\ b \cos v \sin u,\ -c \sin v)$$

$$= (-bc \cos u \sin^2 v,\ -ac \sin u \sin^2 v,\ -ab \cos v \sin v).$$

$$\vec{F} \cdot \vec{dS} = (-z\, ab \cos v \sin v) = (-abc \cos^2 v \sin v)$$

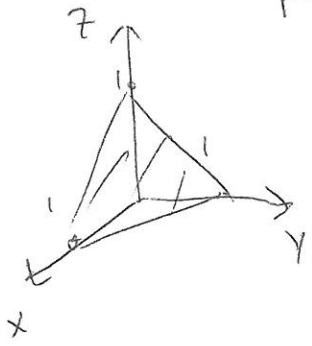
↑ wrong direction!

$$\int_{u=0}^{2\pi} \int_{v=0}^{\pi/2} abc \cos^2 v \sin v$$

$$= \frac{2\pi abc}{3}.$$

core for you.

$\vec{F} = z\hat{k}$.



how to parametrize it?

$x(u,v) = u$

$y(u,v) = v$

$z(u,v) = 1-u-v$.

$$\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} = (1,0,-1) \times (0,1,-1) = (1,1,1)\, du\,dv$$

$\vec{dS}$.

$\vec{F} \cdot \vec{dS} = z = 1-u-v$.

$$\int_{u=0}^{1} \int_{v=0}^{1-u} 1-u-v \, du\,dv = \frac{1}{6}.$$

example

$$\vec{F} = \left\langle ye^{xy} - \frac{y^3}{3},\ x^2y + xe^{xy},\ \sin(xy) \right\rangle$$

flux of curl across upper hemisphere

Stokes ⟶ line integral around unit circle

but involves integrating $(\cos^2 t)e^{\cos t \sin t}$

maple can be used, but best avoided.

$$\operatorname{curl} \vec{F} = (x\cos xy)\hat{i} - (y\cos xy)\hat{j} + (2xy + y^2)\hat{k}$$

on unit disk (S_2): $\operatorname{curl} \vec{F} \cdot \hat{n} = 2xy + y^2$.

$$\iint_{S_2} \operatorname{curl} \vec{F} \cdot \hat{n}\, dA = \iint_{\text{disc}} 2xy + y^2\, dA$$

$$= \int_0^{2\pi} \int_0^1 \left(2(r\cos\theta)(r\sin\theta) + (r\sin\theta)^2\right) r\, dr\, d\theta$$

$$= \cdots = \pi/4.$$

(Start with a Stokes example)

a useful case: integrating a curl $\vec{F}$ over a surface. ~~that's not yet~~ so

we can switch to another surface with the same boundary, and don't even have to ~~change~~ do an extra integral (like when we do the same thing for flux

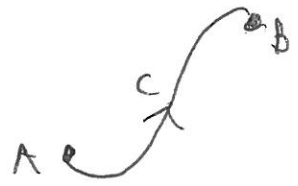
eg. one from previous example.

S_2 = unit circle. $\text{curl}\,\vec{F} = \hat{i}+\hat{j}+\hat{k}$.

$$\iint_{S_2} \text{curl}\,\vec{F}\cdot\hat{n}\,dA = \iint_{S_2} (\hat{i}+\hat{j}+\hat{k})\cdot k\,dA = \iint_{S_2} 1\,dA = \boxed{\pi}.$$

the main thms of integral vector calc.

① fatl thm of line integrals $\int_C \nabla f \cdot d\vec{r} = f(B) - f(A)$



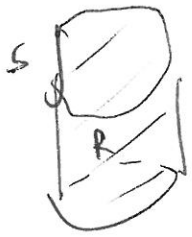
"Boundary of C is A and B".

② Green's thm. $\oint M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$ ($\operatorname{div}\vec{F}$)

"Boundary of R is C"

this week:

③ divergence thm (Gauss's) $\iint_S \vec{F} \cdot \hat{n}\, dA = \iiint_R \operatorname{div}\vec{F}\, dV$.



"boundary of R is S"

S closed.

④ Stokes' thm $\oint_C \vec{F} \cdot d\vec{r} = \iint_S \operatorname{curl}\vec{F} \cdot \hat{n}\, dA$

"boundary of S is C"

"Abe" "to integrate a function, we can integrate some derivative over inside of region".