18.089 Exam 1

Handed out Friday, June 14, 2013; due in class Monday, June 17, 2013

Name:

This exam consists of eight problems, not arranged in any particular order. Please solve all problems in the space provided (or attaching additional sheets as necessary), showing all work as neatly and clearly as possible.

All work must be your own. In particular, you may not seek help from other students in the class or any "live" internet resources. But feel free to use reference works such as your class notes, a textbook, or Wikipedia. If you have any questions about what constitutes an acceptable resource, please ask.

Problem	Value	Score
Problem 1	5	
Problem 2	10	
Problem 3	15	
Problem 4	10	
Problem 5	10	
Problem 6	15	
Problem 7	15	
Problem 8	20	
Total	100	

Problem 1. (5 points) Compute the derivative $\frac{d}{dx} \ln (3^x + 1)$.

Problem 2. (10 points) What is the third derivative of $f(x) = \sin x \cos x$?

Problem 3. (15 points) What is the tangent line to the curve $y = x^3 - 3x^2 + 3x$ at x = 2? Compute the area of the region between the curve and the tangent line.

Problem 4. (10 points) Compute $\int_0^{\pi} x^2 \sin x \, dx$. (Cryptic hint: if once isn't enough, try it again.)

Problem 5. (10 points) Does $\int_1^\infty \frac{1}{x^4+1}$ converge or diverge? Explain.

Problem 6. (15 points) Compute $\int \frac{1}{x^3 + 4x^2 + 5x} dx$.

Problem 7. (15 points) The region above the curve $y = x^2 + 1$ and below the curve $y = -x^2 + 2x + 1$ is rotated around the x-axis. (Just for emphasis: we are rotating around the x-axis, NOT the y-axis.) Compute the volume of the resulting solid.

Problem 8. (20 points) Compute $\int \cos^4 x \, dx$ and $\int \cos^5 x \, dx$.