18.086 Final Project May, 13th, 2014

Numerical investigation of high frequency broadband acoustic backscatter in the Connecticut River Estuary with comparisons to field observations

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Abstract:

Underwater acoustic techniques have been used extensively to remotely sense the interior and floor of the ocean over the last eight decades. The Lavery lab at WHOI (http://www.whoi.edu/hpb/Site.do?id=4132) has specialized in developing high frequency (>10kHz) scattering techniques for sensing biological (zooplankton, small fish) and physical (ie turbulent mixing and stratification) processes. This project builds on these approaches by developing a one dimensional model for scattering from a density interface. Field observations suggest strong density gradients are a common source of scattering in the ocean and estuaries. Despite the common occurrences of these scattering layers there has been limited investigation into what exactly leads to scattering at these sharp interfaces, for example is the scattering truly due to a density gradient, double diffusion, turbulence, particles and or biology in those scattering layers? In this project a scattering layer observed in the Connecticut River Estuary is analyzed and modeled using a one dimensional model of the wave equation for inhomogeneous media. Comparison between observations in the field and the model output are made for the scattering strength of the layer as a function of frequency. The perfectly matched layer (PML) method is used to reduce reflections at the boundaries of the computational domain.

Introduction:

Scattering

Propagation of sound in water is governed by the wave equation. In the ocean gradients of sound speed and density can occur on similar scales of a wavelength associated with frequency of echo sounders that are deployed in the field. Because the wavelength of the pressure waves are on the same scale as the fluctuations in sound speed and density the wave equation for inhomogeneous media, as seen in equation 1, must be used for many problems of oceanographic interest. This is because assuming medium homogeneity and or infinitely thin gradients cannot be accurately done.

$$\nabla \cdot \left(\frac{1}{\rho_o}\nabla p\right) - \kappa\frac{\partial^2 p}{\partial t^2} = S$$

where S is the source term and κ is the compressibility of the medium ρ_o is the zeroth order density field and p is the acoustic pressure. This study focuses on frequencies from 10-600kHz and therefor wave lengths on the scales of 15.0cm-0.25cm. These scales are of the same order as many of the density and sound speed gradients in the ocean. Because of the similarity in scales of the acoustic wave wavelength and the density and sound speed gradients some of the pressure of an incoming acoustic wave is reflected back or scattered back toward the source of the acoustic wave. Note that sound speed and density gradients can arise from both physical processes as well as biological targets (e.g. plankton, fish).

Scattering from microstructure as well as density gradients is considered in this study however only scattering from density gradients is modeled. Density gradients for this study will be considered a density profile which continuously increases or decreases over the layer thickness. An example of this can be seen in figure1a. Microstructure will be considered a density profile where density and sound speed fluctuate within the layer as seen if figure 1b.

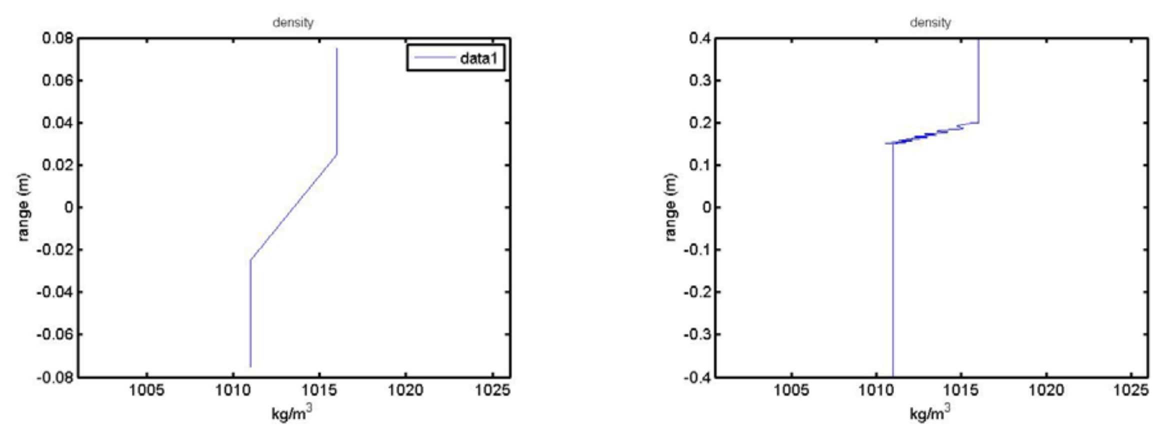


Figure1: figure1a show a density gradient profile where the density (in this case) is continuously increasing with range. Figure 1b shows a profile represetantive of microstucture. Note the strong oscillations in the profile. These oscillation are associated with eddies lifting heavy fluid above lighter fluid creating unstable stratification and hence mixing.

It can be seen that the microstructure profile has much sharper spatial gradients than the density gradient profile. The two different profiles arise from different fluid mechanical processes within the ocean. An example of a layer from an echo sounder time series is given below and is the subject of investigation for this project.

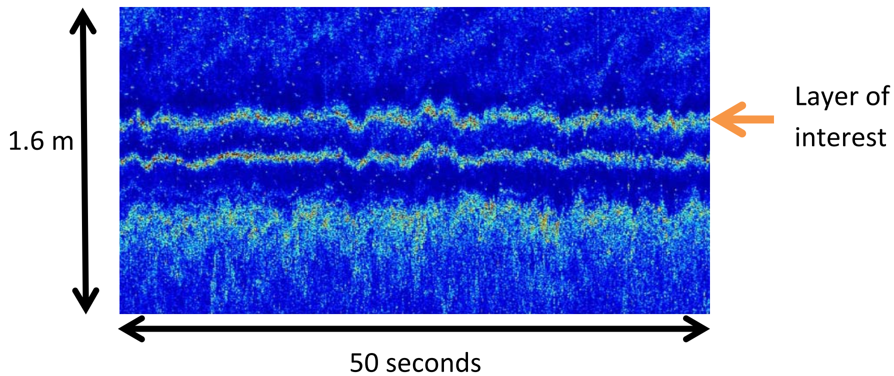


Figure 2: Data from a 500kHz echo sounder looking up from the bottom of the Connecticut River during an experiment in December 2012

Fluid Mechanics

Fluid mechanical processes provide motivation for studying this scattering problem, however, no modeling of the flow field is done for this project so only a brief introduction will be made to the fluid mechanical processes involved with the problem. The primary difference between the two profiles in figure 1 is that figure 1a is representative of a laminar stratified flow and figure 1b is representative of a turbulent stratified flow. Detecting and quantifying turbulence in the ocean is of critical interest in the oceanographic community as turbulence has the ability to mix heat, salt and contaminants ate rates many orders of magnitude larger than molecular diffusion. Better detection allows for better understanding of when where and how much mixing happens in the ocean. Acoustic methods for sensing turbulence are enticing as it allows for rapid and remote sensing of turbulent processes.

Theory:

The Perfectly Matched Layer (PML)

Theoretical formulations of scattering and wave propagation typically assume the target scatterer is in an infinite space and thus any energy traveling away from the object of interest is propagate to infinity and never seen again. Numerically the domain must be truncated as computers have a finite amount of memory and computing power, therefore the boundary conditions of the computational domain become critically important. For simulations of scatterers in free space it is desirable if not necessary for any waves at the boundaries to exit the computation domain without reflecting any energy back into the domain. This is a difficult behavior at the numerical domain boundary to obtain and research is still ongoing in this field. This project utilizes the perfectly matched layer (PML) formulation at the boundary as a way to dissipate incoming waves waves at the boundary.

Consider the first order one dimensional equations for mass (where the equation of state has already been substituted) and momentum for a compressible fluid as given by

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} \quad \text{(momentum)}$$

$$\frac{1}{c^2}\frac{\partial p}{\partial t} = -\rho \frac{\partial u}{\partial x} \ \text{(mass)}$$

where ρ is density, c is soundspeed and u is velocity. Solutions to this set of first order coupled PDE's are of the form

$$p = Ae^{i(kx-\omega t)}$$

where $k = \frac{2\pi}{\lambda}$ and λ is the wavelength and ω is the radian frequency. This solution corresponds to an oscillatory (in time) pressure wave propagating from left right. The PML method does not modify the boundary conditions or the equations of motion to achieve non-reflecting absorption at the boundaries, rather the equations of motion are evaluated in both the real and complex plain very close to the boundary. The effect of having a complex contour x in the solution to the wave equation can be seen in the equation below.

$$p = Ae^{ikRe\{x\}-kIm\{x\}-i\omega t} = Ae^{(ikRe\{x\}-i\omega t)}e^{-kIm\{x\}}$$

Note how the imaginary x component now introduces an exponentially decaying component but no backward propagating component to the solution in the PML which is located along the boundary of the domain and is typically only a few wavelengths long. Therefore the PML exponentially decays any wave that enters into the PML region with no reflection. At the location of domain truncation a rigid boundary condition can be used as any reflection will be exponentially small. This small reflection is okay because it is exponentially small but could be problematic depending on the problem of interest.

Implementation of the PML may seem complicated as it introduces analysis of the wave equation along a complex contour however the PML method can also be viewed as inserting a fictional complex material into the domain rather than analysis of the wave equation along a complex contour. This change in conceptualization allows for the PML to be implemented in real space. To gain a formulation of the PML in real space denote x as

$$\tilde{x} = x + if(x)$$

which allows for the following transformation to be evaluated

$$\partial\tilde{x} = (1 + i\frac{df}{dx})\partial x$$

And therefore

$$\frac{\partial}{\partial \tilde{x}} = \frac{1}{(1 + i\frac{df}{dx})}\frac{\partial}{\partial x}$$

Further simplification can be made by defining $\frac{df}{dx} = \frac{\sigma_x(x)}{\omega}$ which allows for any frequency dependence to be eliminated from the formulation. By taking the Fourier transform of the 1-D wave equation and applying the PML transformation and then transforming back to the time domain the following equations result for waves in the PML.

$$\rho \frac{\partial u}{\partial t} + \rho u \sigma_x(x) = -\frac{\partial p}{\partial x} \quad \text{(momentum)}$$

$$\frac{1}{c^2}\frac{\partial p}{\partial t} + \frac{p\sigma_x(x)}{c^2} = -\rho \frac{\partial u}{\partial x} \quad \text{(mass)}$$

It can easily be seen these set of equation are real valued and a formulation of the PML in real time and space has been found.

Methods:

Numerical scheme

Overview: Simulation of 1-D wave equation for inhomogeneous media is carried out by using second order accurate central differences in space for both first and second derivatives. The solution is stepped in time by implementation of the leap frog method and a fourth order Runge-Kutta step is used to initiate the simulation. A substantial difficulty in simulating the wave equation is dealing with the boundaries of the computation domain.

First the spatial and temporal discretization will be presented for the interior of the computational domain and then the discretization for the PML portions of the domain will be presented. Within the computational domain excluding the PML the finite difference equation for pressure using the leapfrog time stepping method is given by

$$p_i^{n+1} = 2p_i^n - p_i^{n-1} + \frac{\Delta t^2}{\kappa}\left[\left(\frac{D_{i+1}^n - D_{i-1}^n}{2\Delta x}\right)\left(\frac{p_{i+1}^n - p_{i-1}^n}{2\Delta x}\right) + \frac{1}{\rho}\left(\frac{p_{i+1}^n - 2p_i^n + p_{i-1}^n}{\Delta x^2}\right)\right]$$

where $D_i = \frac{1}{\rho_i}$, $\kappa = \frac{1}{\rho c^2}$, i is the space indicie , n is the time step indicie, Δx is the grid spacing and Δt is the time step. Implementation of the PML is slightly more difficult but can be still be implemented as a pressure equation by making use of the acoustic impedance relation for pressure and velocity for plane waves given by

$$\frac{p}{\rho c} = u.$$

Using the plane wave impedance relation is justified in this case because the model is in one dimension only. Making the substitution for velocity in the PML equations yields the following finite difference equation.

$$p_i^{n+1} = \frac{1}{\left[\frac{1}{\Delta t^2} + \sigma_{x_i}\right]}\left[-\left(\frac{2p_i^n - p_i^{n-1}}{\Delta t^2}\right) + \sigma_{x_i}\left(\frac{p_i^{n-1}}{\Delta t}\right) + \rho_i c_i^2 \sigma_{x_i}\left(\frac{U_{i+1}^n - U_{i-1}^n}{2\Delta x}\right) + p_i^n \sigma_{xx_i} + \rho_i c_i^2\left(\frac{D_{i+1}^n - D_{i-1}^n}{2\Delta x}\right)\left(\frac{p_{i+1}^n - p_{i-1}^n}{2\Delta x}\right) + c_i^2\left(\frac{p_{i+1}^n - 2p_i^n + p_{i-1}^n}{\Delta x^2}\right)\right]$$

Where $U_i^n = \frac{p_i^n}{\rho_i c_i^2}$ from using the plane wave impedance relationship and σ_{xx_i} is the second derivative of the attenuation term in the PML. These equations are implemented to simulate scattering from a density interface.

The time step for all simulations was defined as $\frac{\Delta x}{\max(c)}$ and the spatial discretization was chosen to be $\frac{\max(c)}{20 f_{max}}$ where f_{max} is the maximum frequency of interest. The factor of twenty was chosen to avoid numerical dispersion in the simulation. Using these parameters proved to work well for and no issues of stability and or convergence were found.

Results:

Model Verification

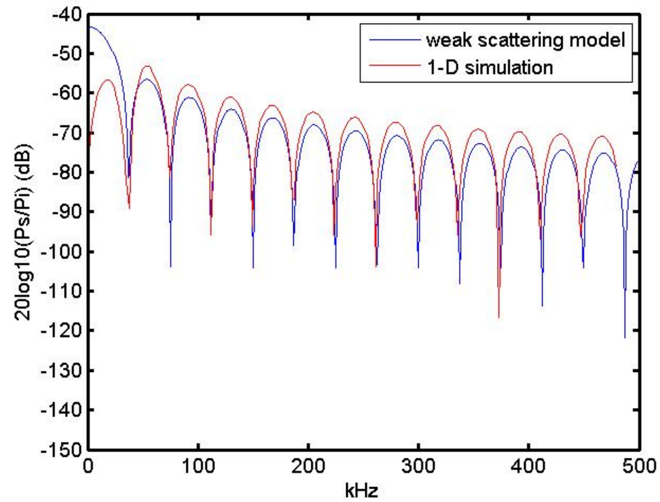
Comparison of the test model with an analytical model that assumes weak scattering was done as well as a test of the model accuracy to check the validity of the code. Table1 one shows the results of increasing the resolution of the model and the error compared to an exact solution. In this case an interface of zero thickness was modeled. It can be seen the model is second order accurate if the exponentially small reflections are ignored in the calculations.

N	R_theory	R	error	order
1000	0.0385	0.043	0.0044	
2000	0.0385	3.95E-02	1.00E-03	2.14E+00
4000	0.0385	3.88E-02	2.26E-04	2.15E+00
8000	0.0385	0.0386	5.06E-05	2.16E+00

Table1: shows the results of the accuracy test of the model. It can be seen the model is second order accurate. Note it will become less accurate if the small reflections from the PML are included.

The weak scattering model is taken from Lavery and Ross 2007 and is known to slightly under predict the amplitude of scatter for linear or sharp interfaces. A sharp interface was used for this comparison.

Figure 3: Shows the scattering amplitude as a function of frequency. Note how the nulls line up very closely and the overall trends of both curves are very similar. The curves are offset by about 4dB which is expected for the density and sound speed profiles used.



The model was also analyzed in the time domain to ensure the absorbing boundaries were working properly. Figure 4 shows the transmitted wave being attenuated on at the end of the computational domain. It can be seen the PML does an effective job attenuating the signal but does reflect a portion of energy back into the domain.

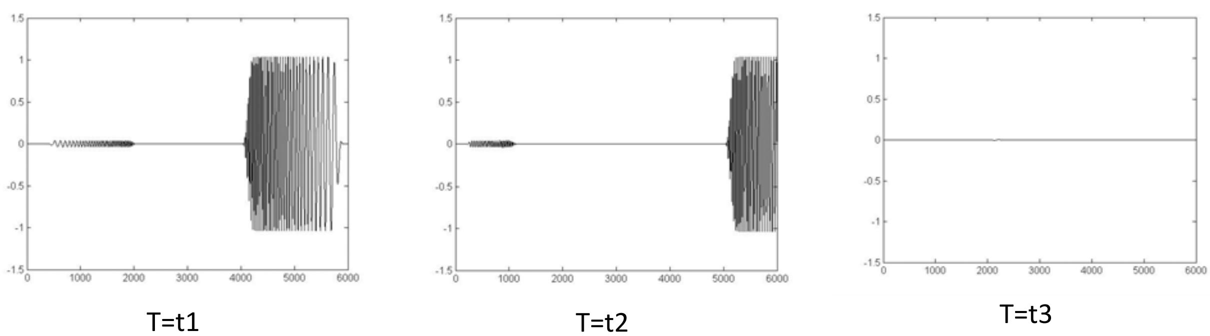


Figure 4: shows time increasing from left to right. Each figure is a snapshot in time. The PML absorption can be clearly seen as there is no large reflection by T=t3.

Models compared to observations

The 1-D model was used to simulate scattering from an interface representative of the interface observed in the field. The weak scattering model was also used to calculate scattering strength as a function of frequency and was compared to observational results. Observations of the density and sound speed structure in the field vastly under-sampled the interface compared to the scales of the acoustic wave length of interest. The CTD is capable of vertical resolution on the order of 10cm and the acoustic wavelength is on the order of 1mm. The large scale structure of the observed density and sound speed profile can be seen in figure 5. A zoomed in plot of the observed profile and the profile used in the model are shown in figure 6. The profile used in the model is based on the observed density and sound speed above and below the layer of interest and the thickness of the interface derived from the acoustic record as seen in figure 7. The thickness of the density interface is deduced from the acoustic record because it is known the CTD under samples the density structure and the acoustic measurement s have a vertical resolution of ~ 1cm which is less than the observed thickness of the interface. The CTD profile provides the values of the density and sound speed that are associated with the model structure.

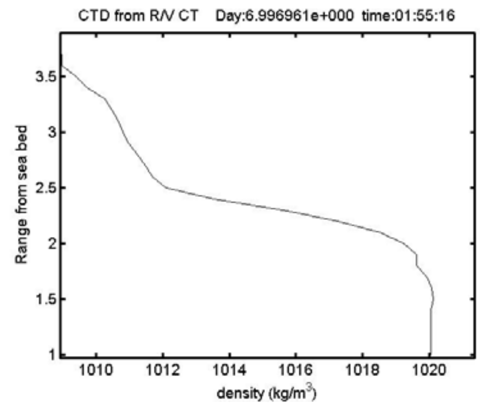


Figure 5: shows the CTD profile the layer is based off of. Note that the CTD vastly under samples the sound speed density structure that is on the scale of the acoustic wave lengths used.

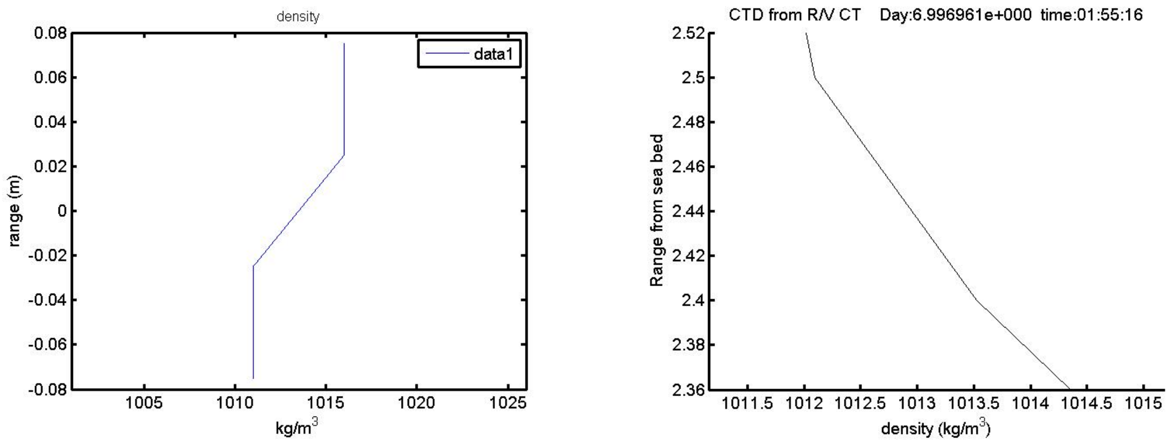


Figure 6: shows the density profile used in the model (left) and the observed (right) density profile. Note the course resolution of the observations. Note the sound speed profiles are assumed to be of identical structure but with different gradients.

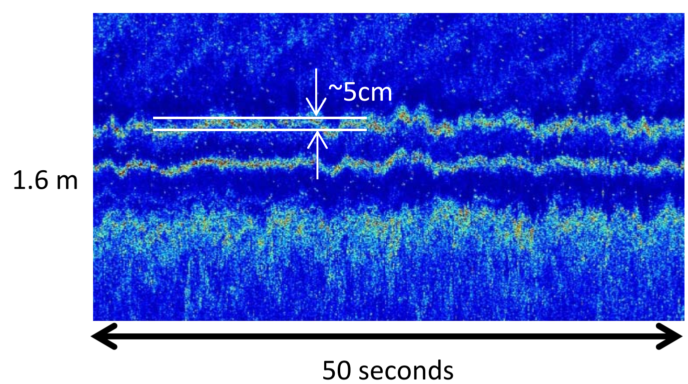


Figure 7: Acoustic Record of the observed interface. The thickness of the interface is derived from this record.

Given the order of magnitude difference in the density and sound speed spatial scales of interest and the spatial scales resolved by the CTD it would seem impossible to make a useful comparison between the observed and modeled scattering. The primary scientific interest of this project, however, is to determine if layer scattering could explain the observed spectral shape and amplitude rather than provide an accurate model description of the scattering processes. Using the procedure discussed above to approximate the density and sound speed profile a comparison of physical relevance is achieved by modeling the layer as a laminar using the acoustic data to infer a thickness and the CTD to inform the change in sound speed and density over the layer.

It can be seen in Figure 8 that the models and the data do not agree in slope or amplitude. Note that the two model outputs are for single ensonifications of the interface. Since these observations are made in a dynamic environment it is assumed the interface will slightly change thickness and that the average of many ensonifications will approach smooth spectra following the peaks of the modeled spectra. It should also be noted that the spectra has a slope that is smaller than expected for scattering from homogeneous isotropic stratified turbulence.

The discrepancies in the observations and the models are significant and scattering levels of -60db could only be explained by the model for an interface of 5mm as seen in figure 9. It can be seen a layer scattering model approaches the correct amplitude but does not have a slope increasing with frequency.

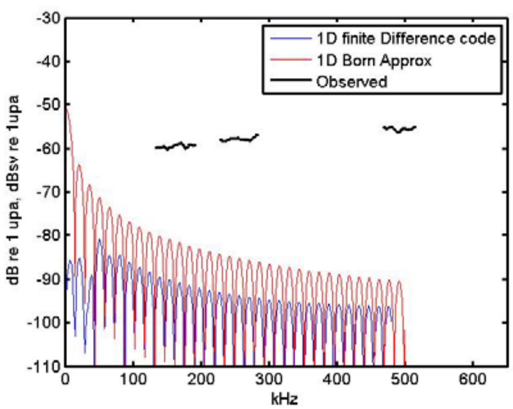


Figure 8: It can be seen there is large discrepancy between the two models and a large difference between the models and the observed spectra

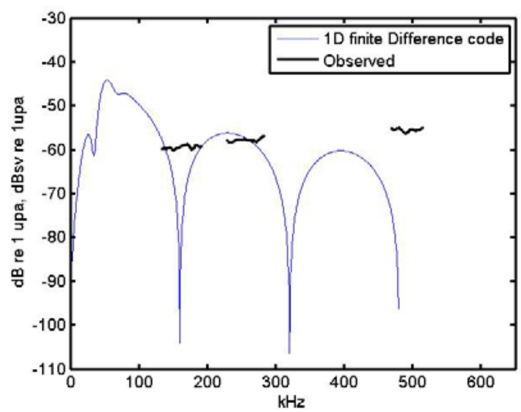


Figure 9: Observed scatter and the 1-D model for an interface 50mm thick, 10 times thinner than what is observed.

If a sound speed and density profile representative of a turbulent stratified flow were used, as seen in figure 10, instead of a density gradient profile the results from the model become more consistent with the observations as seen in figure 11.

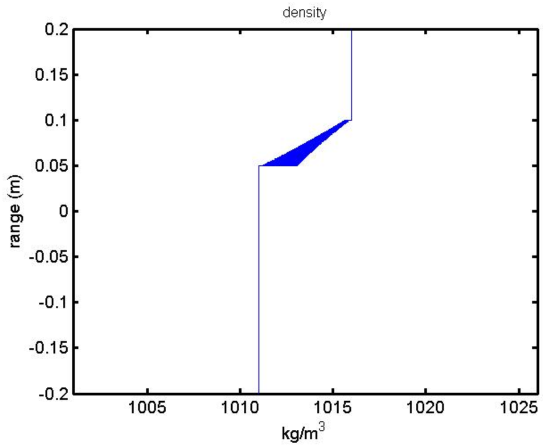


Figure 10: Shows the density profile representative of a turbulent profile.

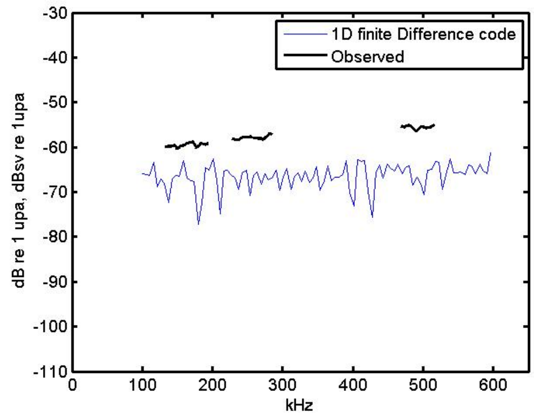


Figure 11: Scattering spectra from the profile. Note that substantial increase in amplitude for the model output

The turbulent profile adds much stronger gradients in sound speed and density. The enhanced gradients increase the scattering amplitude. Figures 10 and 11 are meant to highlight the effect of strong gradients on backscatter amplitude. Note the oscillation wavelengths used to model the turbulence are based on eddy scales expected in the Connecticut River estuary.

Conclusions:

The analysis above indicates scattering from the observed interface is not from a sharp linear sound speed and density gradient. This conclusion is based on estimating the layer thickness from an acoustic record and the density and sound speed change from CTD cast. For the sound speed and density gradient inferred from the data the models predict scattering levels much smaller than the observed levels. Adding many oscillations in the density and sound speed profile, meant to emulate overturning eddies of the size expected in the field, produces scattering at similar levels to the levels observed in the field. This result suggests the scattering could be from turbulent microstructure. The spectra from the field data does not agree well with models for scattering from homogeneous isotropic stratified turbulence which suggests the scattering could be from other sources such as biological scattering, inhomogeneous and or anisotropic turbulence.

Caution should be taken in taking the results of this modeling exercise too far. In the field the scattering volume has 3-D structure, the incident wave is not a perfect plane wave and the ensonified volume changes with frequency.

Characterizing the scattering from layers like this is of critical importance to understanding mixing and mixing rates in the ocean. Layers like this are frequently observed in the field yet there are no models that accurately predict what is giving rise to the scattering. This modeling exercise provides further motivation to develop an accurate scattering model, as it seems clear these thin scattering layers are not sharp interfaces but consist of biological scatters or active mixing. Whatever the true scattering source is it is of significant interest for the oceanographic community to identify it.