MIT Dept. of Mathematics
Benjamin Seibold

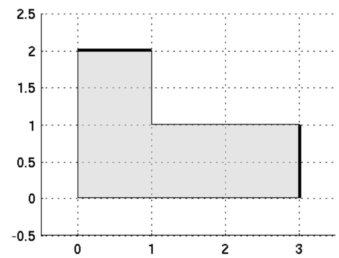
18.086 spring 2008
Exercise Sheet 4

Out Mon 03/31/08 Due Fri 04/18/08

Exercise 9

Consider the domain $\Omega \subset \mathbb{R}^2$, as given in the figure below. The boundary Γ consists of Dirichlet boundaries Γ_D (thin lines) and Neumann boundaries Γ_N (fat lines). Using finite differences, we would like to approximate the Poisson problem

$$\begin{cases} -\Delta u = 1 & \text{in } \Omega \\ u = f & \text{on } \Gamma_D \\ \frac{\partial u}{\partial n} = \frac{\partial f}{\partial n} & \text{on } \Gamma_N \end{cases}$$



where $f(x, y) = x^3 y - x y^3 - \frac{1}{2} x^2$.

Write a Matlab function that, for any given resolution parameter, yields a linear system that discretizes the above Poisson problem.

Then choose a small resolution, a medium resolution and a high resolution, and (try to) solve the linear systems by

1. elimination with reordering
2. Jacobi iteration
3. a simple multigrid method (V-cycle)
4. conjugate gradients

Compare the approaches with respect to accuracy and run times. Up to which resolution can you go with the best method?

Remarks:

- Construct the matrix for interior points only, using lexicographic ordering. View Ω either as a composition or a subtraction of two rectangles. Rectangular geometries can be discretized using Matlab's `kron`.
- For elimination, use Matlab functions. The other three methods have to be implemented by hand (codes from the CSE web page can be used).
- Be aware of the difference between discretization error and errors in solving the

linear system approximately.

- For a small test problem $A\vec{u} = \vec{b}$, the accuracy of a linear solver is best measured by the norm of the error $\|\vec{u}_{\text{approx}} - \vec{u}_{\text{correct}}\|$. However, for high resolutions, this error may not be accessible. In those cases, the residual $\|A\vec{u}_{\text{approx}} - \vec{b}\|$ can be used.
- The high resolution may cause some solution methods to fail. The medium resolution shall be chosen, so that all considered solution methods succeed.